Basic Tools

Characteristic Functions

cadlag, caglad, price processes, trading strategies.

filtrations

Exponential random variables

$$\lambda e^{-\lambda y} 1_{y \geq 0}$$

We can define stack one after another

$$\tau_1 + ... + \tau_n$$

If we define

$$N_t = \inf\{n \geq 1, \sum_{i=1}^{n} \tau_i > t\}$$

then

$$P(N_t = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

So, N_t follows a Poisson distr with parameter λt.

N_t is called a Poisson processes

It has indep increments and the distr of $N_t - N_s$ is the same as the one of the N_{t-s}.

The characteristic function is:

$$E(e^{iuN_t}) = e^{\lambda t(e^{iu}-1)}$$

It is not a martingale (we keep on adding 1s)

The expected value is λt.

Defining a new process as $\tilde{N}_t = N_t - \lambda t$ (the compensated process) gives us a martingale

The Poisson process will be used to model jumps, but we have to modify it (all the jumps are of size 1)

Random Measures

Call $T_1, T_2, ..$ the jump times of a Poisson process

given a set A on the R^+ we can define

$$M(\omega, A) = \#\{i \geq 1, T_i(w) \in A\}$$

It satisfies:

$$E(M(A)) = \lambda |A|$$

M is the derivative of the Poisson process: it is a sum of Dirac deltas.

For any A (measurable) $M(A)$ follows a Poisson distr with parameter $\lambda|A|$.

We can also associate a random measure to the compensated Poisson process.

$$\tilde{M}(\omega, A) = M(\omega, A) - \lambda|A|$$

We can also define a notion of Poisson random measure (without starting with a Poisson process) following these ideas.