Robust Replication of Default Contingent Claims

Peter Carr and Bjorn Flesaker
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Presented by Yoav Tamir and Phil Jones

- Paper is located at:
  http://www.usc.edu/schools/business/FBE/seminars/papers/FMath_10-17-06_CARR_RRDCC3.pdf

- Most of Peter Carr's papers are located at:
  http://math.nyu.edu/research/carrp/

  - Static Hedging of Exotic Options
  - Static Options Replication

- The paper outlines a framework to replicate default-contingent claim by taking static positions in CDS of different maturities;
- Key assumptions – deterministic interest rates and constant recovery rate;
- The motivation is to structure a replication strategy that makes no assumption on the process that triggers default;
- An important output of this process is the ability to extract risk-neutral survival probabilities from a CDS curve.

Objectives:

- Extract unique arbitrage-free prices of a set of default contingent claims;
- Indicate the positions needed to replicate the payoffs of the target claims;
- Key Advantage is low execution risk;
- Disadvantage is that it requires more initial liquidity in CDS than a dynamic method.

Methodology:

- Use two hedging instruments – cash account and Static CDS positions – ***self-financing static hedge***

- At time zero setup a cash account and a static position in CDS;

- Mechanics:

    - Initial CDS positions are never adjusted;

    - Cash flows from CDS and contingent claim are handled by the bank account.

- Starting with a ***Defaultable Annuity [DA]:***
  - Pays one dollar per year until the earlier of a random default or maturity;
- Motivation for using DA:
  - Similar to the payoff from the premium leg of a CDS;
  - The solution of the replication problem for a DA can be used to determine the portfolio weights when replicating an arbitrary default-contingent claim;
- Recovery Rate – assumed known and constant.

Replication via Backward Equation:

- Setup:
  - At inception investor is selling CDS and establishing a bank account;
  - The bank account is used to pay the claims on the CDS
- Notations:
  - $M(t)$ – Bank account;
  - L – Loss given default on a bond
  - $R(t)$ – Recovery rate of claim
  - $S(t)$ – Premium paid on credit derivative

- Two equations: *Recovery Matching Condition* and *Self-financing Condition;*

- Recovery Matching Condition – funds in the bank account minus losses from default equal the recovery value. In essence M(t) and Q(t) are set to equal R(t):

   - $$M(t) - L * \int_t^T Q(u)du = R(t)$$

- Self-financing Condition – Change in the bank account is driven by interest rate payments, premium payments on CDS and interest payment on the default-contingent claim:

   - $$M'(t) = r(t) * M(t) + \int_t^T S(u) * Q(u)du - c(t)$$

- Combining the two conditions:
  - $$M''(t) - \left[r(t) + \frac{S_0(t)}{L} \right] * M'(t) - r'(t) * M(t) = f(t)$$
  - $$f(t) \equiv c'(t) - \frac{S_0(t)}{L} * R'(t)$$
- Two terminal conditions:
  - M(T) = 0
  - $$Lim_{t \uparrow T} M'(t) = -c(T)$$

- Example – Flat CDS Curve:

  - Assuming: c(t)=1, R(t)=0 and S(t) = S_0;

  - $$\frac{\partial}{\partial t}\left\{\frac{\partial}{\partial t}M(t)-\left[r(t)+\frac{S_0}{L}\right]*M(t)\right\}=0$$

  - Adding terminal conditions above;

  - Survival Contingent Bank Balance:

    - $$M(t)=\int_t^T e^{-[y(t;t')+\frac{S_0}{L}](t'-t)}dt'$$

- Where: $y(t;t') \equiv \dfrac{\int_t^{t'} r(v)dv}{t'-t}$ Is the yield to maturity of a default-free bond

- And Q(t) is:

- $$Q(t) = \frac{1-[r(t)+\frac{S_0}{L}]*e^{-[y(t;t')+\frac{S_0}{L}](t'-t)}}{L}$$

Example Flat Forward Curve:

- Assuming: c(t)=1, R(t)=0 and r(t) = r;
- $$\frac{\partial^2}{\partial t^2} M(t) - \left[r + \frac{S_0(t)}{L} \right] \frac{\partial}{\partial t} M(t) = 0$$
- Adding terminal conditions above;
- Survival Contingent Bank Balance:
  - $$M(t) = \int_t^T e^{-\int_u^T \left[r + \frac{S_0(v)}{L} \right] dv} du$$

- And Q(t) is:

- $$Q(t) = -\frac{e^{-\int_u^T \left[r + \frac{S_0(v)}{L} \right] dv}}{L}$$