Principal Components Analysis in Yield-Curve Modeling

Carlos F. Tolmasky

April 4, 2007

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    - Interest rates.
    - Commodities.
- Does it make sense to model each underlying individually?

Front Month Crude

Crude Curve

<HELP> for explanation. N127 Comdty CT

<PAGE> now scrolls 17 contracts. Enter # <GO> to scroll contracts.

Session: PIT Contract Table

WTI CRUDE FUTURE

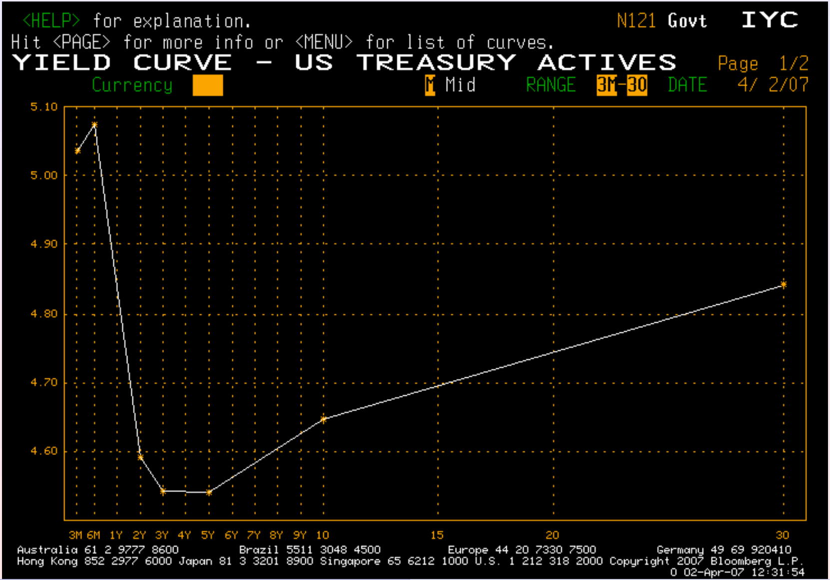
Exchange Web Page Pricing Date: 3/30/07

New York Mercantile Exchange --AS REPORTED 3/30 -- 2

Grey date = options trading

↓ Scroll		Last	1 Change	Time	Bid 1	Ask	OpenInt (1354924)	TotVol (137888)	Previous Close
1) CLK7	May07	65.87s	-.16	Close			358859	28289	66.03
2) CLM7	Jun07	67.47s	+.19	Close			192438	28833	67.28
3) CLN7	Jul07	68.25s	+.20	Close	67.70		75684	7929	68.05
4) CLQ7	Aug07	68.73s	+.12	Close			34944	4473	68.61
5) CLU7	Sep07	69.08s	+.07	Close			39717	3358	69.01
6) CLV7	Oct07	69.35s	+.03	Close			29727	3295	69.32
7) CLX7	Nov07	69.56s	+.03	Close			25285	2278	69.53
8) CLZ7	Dec07	69.71s	+.03	Close			146037	14094	69.68
9) CLF8	Jan08	69.81s	+.02	Close			35615	3429	69.79
10) CLG8	Feb08	69.89s	+.01	Close			13916	435	69.88
11) CLH8	Mar08	69.94s	unch	Close			14852	4500	69.94
12) CLJ8	Apr08	69.99s	unch	Close			11765	70	69.99
13) CLK8	May08	70.04s	unch	Close			9966	100	70.04
14) CLM8	Jun08	70.09s	unch	Close			42902	3698	70.09
15) CLN8	Jul08	70.08s	unch	Close			5359	300	70.08
16) CLQ8	Aug08	70.04s	-.01	Close			6099	950	70.05
17) CLU8	Sep08	70.00s	-.02	Close			14037	400	70.02

Yield Curve


<HELP> for explanation.
N121 Govt IYC
Hit <PAGE> for more info or <MENU> for list of curves.
YIELD CURVE - US TREASURY ACTIVES
Page 1/2
Currency
M Mid
RANGE 3M-30
DATE 4/ 2/07
5.10
5.00
4.90
4.80
4.70
4.60
3M 6M 1Y 2Y 3Y 4Y 5Y 6Y 7Y 8Y 9Y 10
15
20
30
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2007 Bloomberg L.P.
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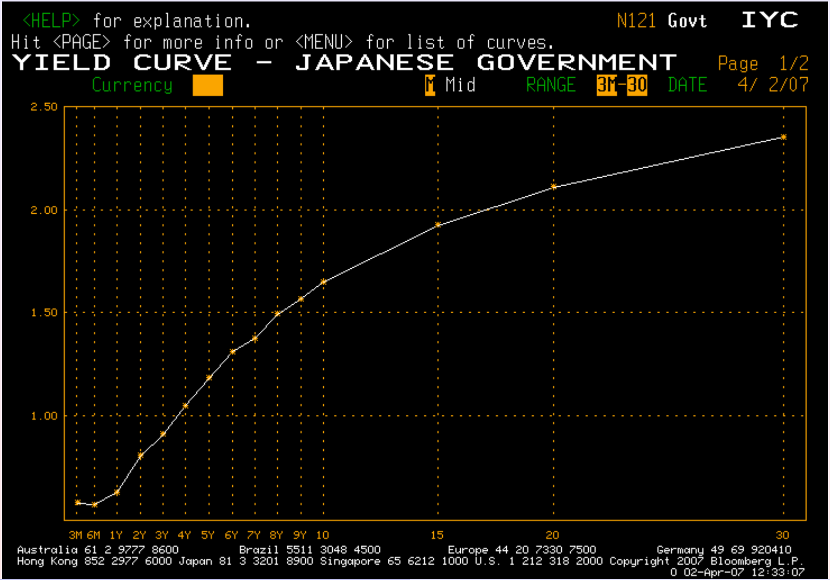
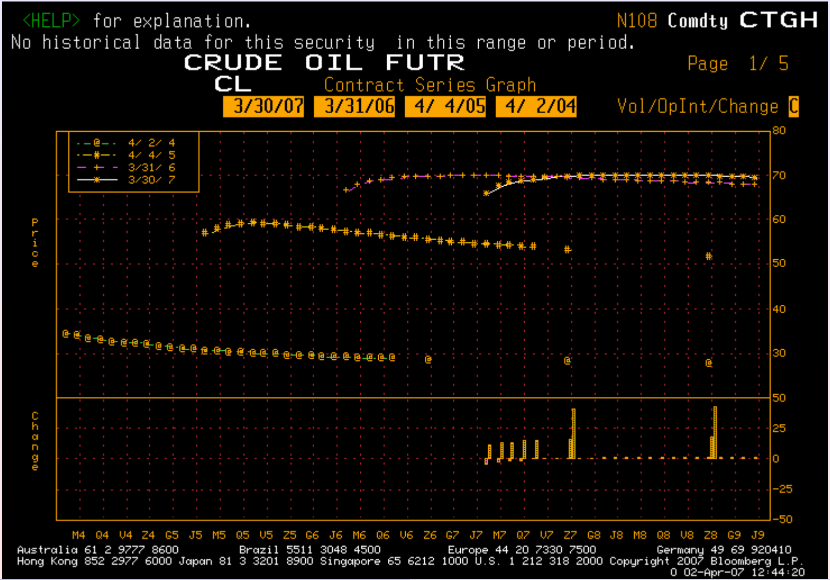
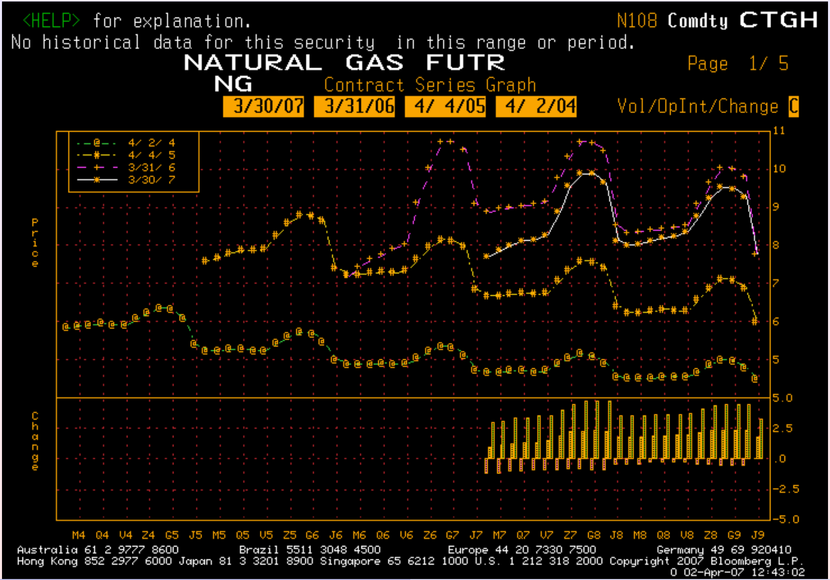
Japanese Yield Curve



Crude Curve through time



Natural Gas Curve through time



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But, how do we choose the $\sigma_{j,i}$??

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- Loads (or lots?) of other people report the same kind of results in many other markets.

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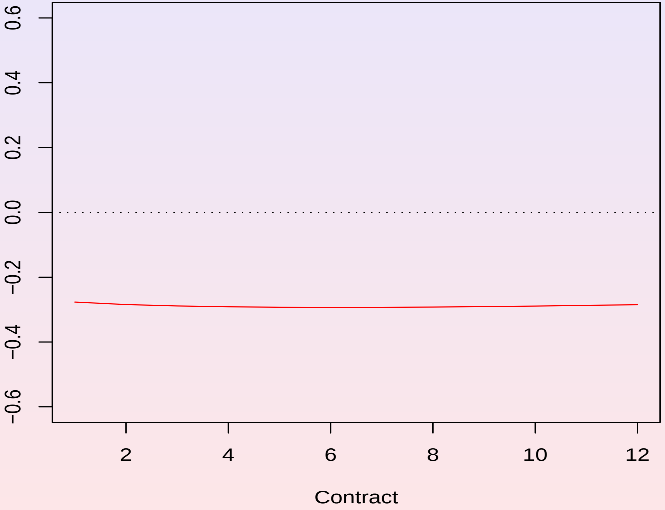
  - Some spillover effects found.

Table: Correlation Matrix for Changes of the First 12 Crude Oil Futures Prices

1.000	0.992	0.980	0.966	0.951	0.936	0.922	0.08	0.892	0.877	0.860	0.848
0.992	1.000	0.996	0.988	0.978	0.966	0.954	0.941	0.927	0.913	0.898	0.886
0.980	0.996	1.000	0.997	0.991	0.982	0.973	0.963	0.951	0.939	0.925	0.914
0.966	0.988	0.997	1.000	0.998	0.993	0.986	0.978	0.968	0.958	0.946	0.936
0.951	0.978	0.991	0.998	1.000	0.998	0.994	0.989	0.981	0.972	0.963	0.954
0.936	0.966	0.982	0.993	0.998	1.000	0.999	0.995	0.90	0.983	0.975	0.967
0.922	0.954	0.973	0.986	0.994	0.999	1.000	0.999	0.996	0.991	0.984	0.978
0.08	0.941	0.963	0.978	0.989	0.995	0.999	1.000	0.999	0.996	0.991	0.985
0.892	0.927	0.951	0.968	0.981	0.90	0.996	0.999	1.000	0.999	0.995	0.991
0.877	0.913	0.939	0.958	0.972	0.983	0.991	0.996	0.999	1.000	0.998	0.996
0.860	0.898	0.925	0.946	0.963	0.975	0.984	0.991	0.995	0.998	1.000	0.998
0.848	0.886	0.914	0.936	0.954	0.967	0.978	0.985	0.991	0.996	0.998	1.000

First four eigenvectors for oil



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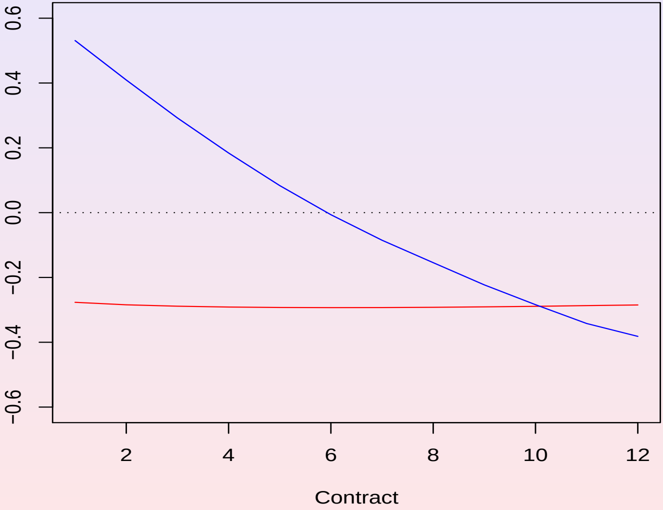

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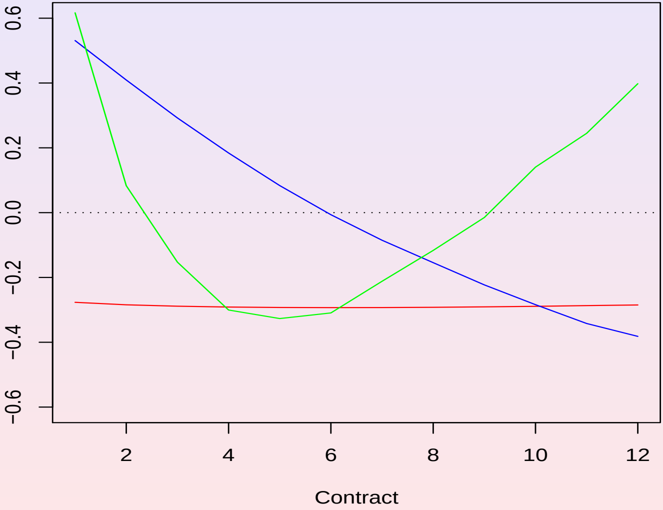

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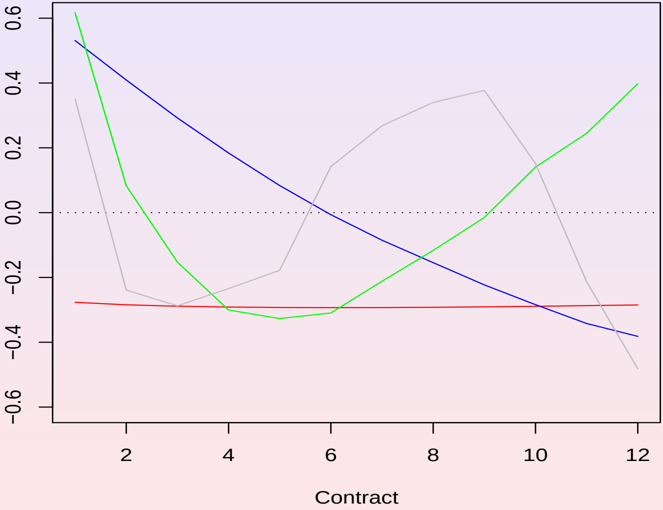

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- Why is the result "market-invariant"?
- Because all the correlation matrices are very similar.
- They all look like $\rho^{|i-j|}$ with ρ close to 1.
- Proved that the eigenvectors of those matrices converge to $cos(nx)$ when $\rho \to 1$.

Correlation matrix:

$$\begin{pmatrix} 1 & \rho^{\frac{T}{n}} & \rho^{2\frac{T}{n}} & \dots & \dots & \rho^{n\frac{T}{n}} \\ \rho^{\frac{T}{n}} & 1 & \rho^{\frac{T}{n}} & \dots & \dots & \rho^{(n-1)\frac{T}{n}} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \rho^{(n-1)\frac{T}{n}} & \rho^{(n-2)\frac{T}{n}} & \rho^{(n-3)\frac{T}{n}} & \dots & 1 & \rho^{\frac{T}{n}} \\ \rho^{n\frac{T}{n}} & \rho^{(n-1)\frac{T}{n}} & \rho^{(n-2)\frac{T}{n}} & \dots & \rho^{\frac{T}{n}} & 1 \end{pmatrix}$$

or, as an operator:

$$K_\rho f(x) = \int_0^T \rho^{|y-x|} f(y)dy. \tag{1}$$

- A big part of the correlation structure is given by:

$$R(t, T_1)T_1 = R(t, T_0)T_0 + f(t, T_0, T_1)(T_1 - T_0)$$

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- Even if we generate independent forwards we find structure in the correlation matrix of the zeros.
- Looked at the PCAs of fwds in various markets, found nothing interesting.

Alexander-Lvov (2003)

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- Found that this choice is crucial to the correlation structure obtained.
- Could Lekkos' critique be just a matter of the choice of the fitting technique?

- They ask the question:

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  Can we characterize "level-slope-curvature"?

- They look at sign changes in the eigenvectors.
- "Level" means no sign changes.
- This is solved by Perron's theorem.

Lord, Pessler (2005)

Perron's Theorem:

Let A be an N $\times$ N matrix, all of whose elements are strictly positive. Then A has a positive eigenvalue of algebraic multiplicity equal to 1, which is strictly greater in modulus than all other eigenvalues of A. Furthermore, the unique (up to multiplication by a non-zero constant) associated eigenvector may be chosen so that all its components are strictly positive.

- A square matrix A is said to be totally positive (TP) when for all p-uples n, m and $p \leq N$, the matrix formed by the elements a_{n_i,m_j} has nonnegative determinant.

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- If those dets are strictly positive they are called strictly totally positive (STP).
- This is all classical stuff in matrix theory.
- In 1937 Gantmacher and Kreĭn proved a theorem for ST matrices.

Lord-Pessler (2005)

Sign-change pattern in STPk matrices

Assume Σ is an N $\times$ N positive definite symmetric matrix (i.e. a valid covariance matrix) that is STP_k. Then we have $\lambda_1 > \lambda_2 > ... > \lambda_k > \lambda_{k+1} \geq ...\lambda_N > 0$, i.e. at least the first k eigenvalues are simple. Moreover denoting the jth eigenvector by x_j, we have that x_j crosses the zero $j-1$ times for $j = 1, ..., k$.

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- Therefore $STP_3 \Rightarrow$ "level-slope-curvature".
- Condition can be relaxed.
- Definition: A matrix is called oscillatory if it is TP_k and some power of it is STP_k.
- Sufficient condition can be relaxed to being oscillatory of order 3 (actually to having a power which is).

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- Actually that is not true in reality. The diagonals increase in size.
- In modeling correlations Schoenmakers-Coffey proposed a family of matrices that takes this fact into account.
- Lord-Pessler show that these matrices are oscillatory.

Lord-Pessler (2005). Conjecture

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Lord-Pessler (2005). Conjecture

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- $\rho_{i,j+1} \leq \rho_{i,j}$ for $j \geq i$.

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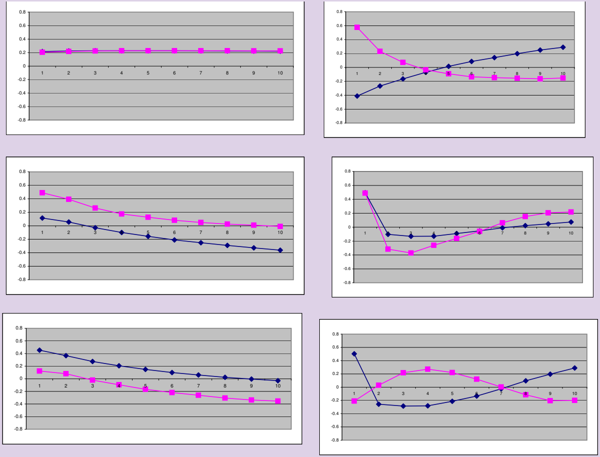
- We can model something like this by assuming a constant correlation intercurve and a different, also constant, correlation intracurve.
- Depending on how high is the intercurve correlation we will get "separation" vectors of different orders.

PCA of crude and heating oil together



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where

$$\begin{pmatrix} 1 & \rho & \rho^2 & \dots & \dots & \rho^n \\ \rho & 1 & \rho & \dots & \dots & \rho^{n-1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \dots & 1 & \rho \\ \rho^n & \rho^{n-1} & \rho^{n-2} & \dots & \rho & 1 \end{pmatrix}$$

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So, depending on the size of the intercurve correlation we will get different order of importance between common frequencies and separating frequencies.

Seasonality in the Eigenvalues (o=heating oil, x=crude)