PROBLEMS IN PRACTICE TEST 2

$$\begin{aligned} y' + xy &= x \\ y(0) &= -1 \end{aligned}$$

44. If y is a real-valued function defined on the real line satisfying the initial value problem above, then $\lim_{x \to -\infty} [y(x)] =$

(A) 0
(B) 1
(C) -1
(D) ∞
(E) $-\infty$

---

54. Choose a real number x uniformly at random in the interval $[0, 3]$. Choose a real number y independently of x, and uniformly at random in the interval $[0, 4]$. Find the probability that $x < y$.

(A) 1/2
(B) 7/12
(C) 5/8
(D) 2/3
(E) 3/4

Date: Printout date: November 10, 2015.

61. A tank initially contains a salt solution of 3 grams of salt dissolved in 100 liters of water. A salt solution containing 0.02 grams of salt per liter of water is sprayed into the tank at a rate of 4 liters per minute. The sprayed solution is continually mixed with the salt solution in the tank, and the mixture flows out of the tank at a rate of 4 liters per minute. If the mixing is instantaneous, how many grams of salt are in the tank after 100 minutes have elapsed?

(A) 2
(B) $2 - e^{-2}$
(C) $2 + e^{-2}$
(D) $2 - e^{-4}$
(E) $2 + e^{-4}$

65. Let g be a differentiable function of two real variables, and let f be the function of a complex variable z defined by

$$f(z) = e^x \sin y + i \cdot (g(x, y)),$$

where x and y are the real and imaginary parts of z, respectively. If f is an analytic function on the complex plane, then $(g(3, 2)) - (g(1, 2)) =$

(A) e^2
(B) $e^2((\sin 3) - (\sin 1))$
(C) $e^2((\cos 3) - (\cos 1))$
(D) $e - e^3(\sin 2)$
(E) $(e - e^3)(\cos 2)$