

- * Check whether two vectors are linear independent or not.
- * Given two points on the Euclidean Plane and determine whether they lie in a square with unit area.
- * Find out the maximum points of the following function:
 $e^{-x} \sin \pi x, 0 < x < 10$
- * What is the minimum value of
 $a+b+c+d$
when the following identity satisfies:
 $4a = 3b = 5c = 15d$
- * If the mean of $a < b < c < d$ is 100, what is the minimum value of $a+d$.
- * It is known that $f'' > 0$ and $f(x) = f(-x)$. Which of the following statements are correct?
 $f(0) < f(1), f(4) - f(3) < f(6) - f(5), f(-2) < \frac{f(-3) + f(-1)}{2}$
- * Suppose there is a field of 8 elements. Determine the number of invertible matrix on this field.
- * f_n and f are continuous functions. $f_n(x) \rightarrow f(x)$ pointwise. Which of the following is/are correct?
 - $\int_0^x F_n(t) dt \rightarrow \int_0^x F(t) dt$
 - $F'_n(x) \rightarrow F'(x)$
 - $\int_0^x f_n(t) dt \rightarrow \int_0^x f(t) dt$
- * Which person made a great contribution to modern algebra/analysis.
- * Determine the number of the trees of the following graph.



* If $g \circ f$ is constant, then which of the following is/are constant?

- I. $f \circ g$
- II. f
- III. g

* $z \in \mathbb{C}$, $\max |z^2 - iz| = ?$

* Which of the following is convergent?

A. $\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$

B. $\int_0^\infty \left(\frac{\cos x}{x}\right)^2 dx$

C. $\int_0^\infty \left(\frac{e^x}{x}\right)^{1/2} dx$

D. $\int_0^\infty \left(\frac{1}{1+x^2}\right)^{1/2} dx$

E. $\int_0^\infty \left(\frac{1-e^{-x}}{x}\right) dx$

* $\prod_{k=1}^7 (z - e^{-k\pi i}) = ?$

* B is closed, A is open, then $B \setminus A$ is

A. open

B. closed

C. neither open nor closed

D. bounded

E. finite

* $\int_0^1 \sqrt{e^x + 2 + e^{-x}} dx$

* $\sum_{n=0}^{\infty} \frac{(-1)^n (n+1)}{n!}$

* The remainder of $\int_0^{4\pi/3} \frac{x^5}{7} dx$

* $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$

* Suppose X and Y be metric space, $A \subseteq X$, f be a continuous map from A to Y , then

I. A is compact, then $f(A)$ is compact

II. If A is bounded, then $f(A)$ is uniformly ~~continuous~~ continuous

III. Given f is bijection, A is bounded, $f(A)$ is compact, then f^{-1} must be continuous.

* Let + and \cdot represent the normal addition and multiplication on with real number. Choose the following which is not monoid.

- A. $(\mathbb{R}, +)$ B. $(\mathbb{R} - \{0\}, \cdot)$ C. $(\mathbb{R} - \{0\}, +)$

* $y = \sin^2(\pi x) \cdot e^x$, $0 < x < 10$, find the local maximum.

* Function f is a function with two linear parts.

$f(0) = f(2) = 0$, $0 < x < 1$, $\max(f) = 1$. Choose the range of f ?

- A. $(2\sqrt{2}, \infty)$
B. $(2\sqrt{2}, \infty]$
C. $[2\sqrt{2}, \infty)$
D. $[2\sqrt{2}, \infty]$

* Which letter is not homeomorphic to the letter C?

- A. J B. N C. S D. O E. U

* Which one is the graph of $y = f(x)$ that satisfies $\frac{dy}{dx} \leq \sin x$

* How many real numbers satisfy the equation $x^5 + 8x - 7 = 0$

* $h(x) = \int_0^{x^2} e^{t+x} dt$, $h'(1) = ?$

* $(1+i)^{10}$

* $\int_0^{\infty} \frac{e^{ax} - e^{bx}}{(1+e^{ax})(1+e^{bx})} dx$

* $C = \{e^{i\theta} : 0 \leq \theta \leq \pi\}$; $\int_C (4z^2 + 3z^2 + 4z^3) dz = ?$

* $a, b \in$ group G . Both a and b have finite orders.

I. If $ab = ba$, then ab has finite order.

II. If ab has finite order, then ba has finite order

III. If ab has finite order, then $a^{-1}b^{-1}$ has finite order.

Which of them are correct?

* What is the smallest positive integer that can be written as $7x + 42y$ where $x, y \in \mathbb{Z}$.

* p is a prime, G is a cyclic group with order p^2 .
How many subgroups does G have.

* Find b such that $f(x)$ is a probability density function

$$f(x) = \begin{cases} 1/(x \ln b x)^2, & x \geq 2 \\ 0, & x < 2 \end{cases}$$

* Sketch the graph of $y^4 + y^3 = 0$

* Suppose A be a connected subset of topology space. Which of the following must be connected:

- I. the interior of A ;
- II. the closure of A
- III. the complement of A .

* Ten questions in all. If four of the first five questions must be answered at least, then how many way are there to answer seven of all the ten questions?

* Fair coins are tossed and when either four consecutive heads or tails appear the process will be stopped.

What is the probability of two consecutive head or tail or any one of them in one row?

* T is a one-to-one and onto mapping defined as:

$$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

which ~~not~~ of the following is/are correct?

$$T \circ (S \circ N) = (T \circ S) \circ N$$

$$T \circ N = N \circ T$$

$$T \circ A = T \circ B \Rightarrow A = B$$

* Choose the graph of

$$\frac{dy}{dx} = \sin y$$

* Check the consistency of the linear equations.

* Function f is a linear function composed of two linear pieces and continuous on $[0, 2]$.

$$f(0) = f(2) = 0 \text{ and } \max f = 1.$$

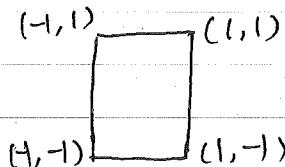
What is the length of the graph of f

* $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Calculate

$$I + \sum_{n=0}^{10} \frac{t^n}{n!} A^n$$

* Calculate the volume of the solid generated by rotated region $\{x=1, y=2, y=x\}$ along the y -axis.

$$\star \int_C \vec{F} \cdot \vec{T} ds = ?$$



Where $F = x - y$ and C is the following circle (counterclockwise)