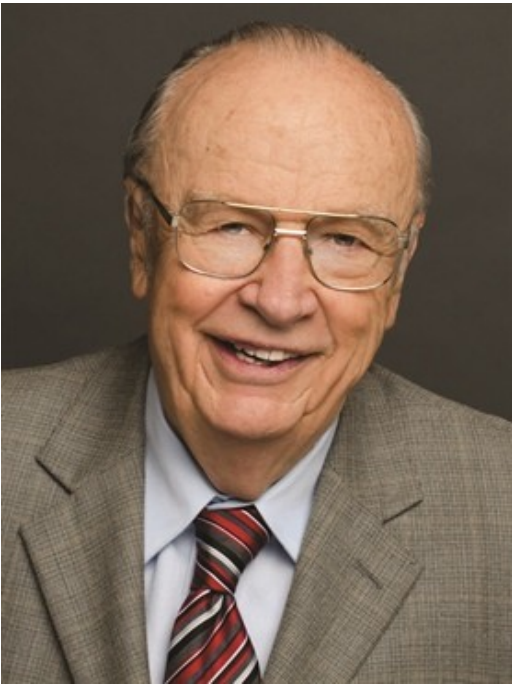


Remembering Ivo Babuška

By [Douglas N. Arnold](#)



Ivo Milan Babuška, 1926-2023. Photo courtesy of the Oden Institute for Computational Engineering and Sciences.

Ivo Milan Babuška, a Czech-American mathematician and engineer who came of age at the end of the precomputer era, passed away on April 12, 2023. He was 97 years old. As one of the creators of the field of computational science and engineering, Ivo was the preeminent authority on the finite element method (FEM). He was thoroughly dedicated to his research and helped develop FEM into an accurate and reliable methodology for computer simulation. Ivo pursued every project with a remarkable level of curiosity, energy, and discipline and generated a torrent of original, insightful, and high-impact contributions.

Czechoslovakia: The Formative Years

Ivo was born on March 22, 1926 in Prague, Czechoslovakia. When he was 13, Ivo's life was turned upside down by a momentous political upheaval — the first of three times this was to occur. Nazi Germany invaded Czechoslovakia and occupied it until the end of World War II in 1945. The Babuška family lost almost everything and the Nazis closed the universities, sabotaging Ivo's educational plans. Despite the family's hardships, Ivo's father engaged a first-rate mathematics professor to tutor his son in university-level mathematics.

When the war ended and universities reopened, Ivo enrolled in the Czech Technical University to study civil engineering, obtaining his first doctorate in 1951. While he was completing this degree, Eduard Čech—a renowned geometer and director of the Mathematics Institute of the Czechoslovak Academy of Sciences—awarded a fellowship for study at the Institute to Ivo and 11 other promising university graduates. Čech sought to revitalize Czech mathematics and guided Ivo toward the rapidly evolving areas of numerical analysis and applied mathematics. Under Čech's skillful guidance, the Institute soon became home to a close-knit, congenial band of brilliant students and professors who not only studied but also socialized together. Ivo later noted the influence that this extraordinary educational environment had on his own future interactions with students and mentees. In 1955, he earned his second Ph.D. — this time from the Mathematics Institute.

Unfortunately, the restoration of democratic institutions in Czechoslovakia was short-lived. In a second life-altering political upheaval, the Soviet-backed Communist Party of Czechoslovakia staged a coup d'état in 1948. They took complete control of the government and quickly converted the country into a satellite state of Stalin's Soviet Union. It was against this dark background that Ivo finished his formal education and launched his career. Luckily, government officials recognized applied mathematics' necessity to industrial development and allowed the field to progress with minimal interference.

A formative experience in Ivo's education was his role on an interdisciplinary team of civil engineers, materials scientists, and mathematicians who—beginning in 1954—consulted on the six-year construction of the Orlik Dam on the Vltava River. The design process raised many technical challenges, such as determining which construction procedures would likely prevent the formation of harmful cracks in the concrete. Ivo led the mathematical efforts by formulating models based on partial differential equations (PDEs), analyzing them mathematically, and studying them further with numerical computation. Since Czechoslovakia was still in the pre-computer era, a few desk calculator operators carried out the numerical computations under Ivo's direction. The efforts of Ivo and the Orlik research team expedited the construction of the dam, which is still in use today. Ivo's lifelong devotion to interdisciplinary science began at this time.

Upon completing his second Ph.D. in 1955, Ivo was appointed head of the Department of Constructive Methods of Mathematical Analysis within the Mathematical Institute of the Czechoslovak Academy of Sciences — a position he held until he left Czechoslovakia. He published prolifically, and his reputation as a numerical analyst and applied mathematician steadily grew. In 1956, Ivo founded the journal *Aplikace Matematiky* (now known by its English title, *Applications of Mathematics*) and served as its first editor-in-chief. He was granted a Doctor of Science degree in 1960 and received the Czechoslovak State Medal—the nation's highest scientific award—in 1968. In 1964 and 1967, Ivo organized the International Conference on Basic Problems of Numerical Analysis in Czechoslovakia. As a reflection of his growing international reputation, top numerical analysts from around the world—including Sergei Sobolev, Andrey Tikhonov, Gene Golub, Eugene Isaacson, Olof Widlund, Lothar Collatz, and James H. Wilkinson—were in attendance.



Attendees of the 1967 International Conference on Basic Problems of Numerical Analysis, which took place at Liblice Castle in Czechoslovakia, gather for a group photo. Ivo Babuška organized both the 1964 and 1967 iterations of this meeting, which attracted some of the top numerical analysts from around the world. Numbered from left to right: (1) Ivo Babuška, (2) Lothar Collatz, (3) Andrey Tikhonov, (4) Bert Hubbard, (5), Hans Jörg Stetter, (6) James H. Wilkinson, and (7) Frank W.J. Olver. Photo courtesy of Douglas Arnold.

The third and most consequential political upheaval for Ivo came in 1968, when the Soviet Union brutally suppressed the Prague Spring reform movement. In August of that year, hundreds of thousands of Soviet and Warsaw Pact troops stormed Czechoslovakia. Fortuitously, Ivo already had plans underway for a one-year visiting professorship at the University of Maryland (UMD) and managed to obtain the necessary documentation to travel by ship to the U.S. with his wife Renata and their young children, Lenka and Vit. The family arrived in Maryland in September 1968.

Maryland: Finding His Life's Work in America

Ivo's visiting appointment at UMD was in the Institute for Fluid Dynamics and Applied Mathematics (IFDAM, later the Institute for Physical Science and Technology). At the end of the year, he accepted a permanent position as a distinguished professor at IFDAM. He remained there for 27 years.

Ivo's research underwent a sharp discontinuity when he emigrated to Maryland. In Czechoslovakia, he had published more than 40 mathematical papers; not one of them even mentioned FEM. After arriving in the U.S., Ivo published roughly 300 more papers. Starting with the very first, virtually every one addressed finite elements; the future *ne plus ultra* of FEM had found his true calling and never looked back. In the next five decades, Ivo continued to produce significant and often seminal results in rapid succession. This work—along with his powerful personality, generous nature, and leadership qualities—made him a driving force in the development of FEM.

In 1970, Ivo and a few colleagues conceived of the need for regular short meetings in the burgeoning area of finite elements. They envisioned an unusual format that did not specify speakers in advance but instead allowed any participant with relevant work to present. To match this informal structure, the group selected a genial name for the conference series: the [Finite Element Circus](#). The first Circus took place at UMD in 1970; future iterations were held at the University of Chicago, Harvard University, and Cornell University. Ivo served as Circus Ringmaster for the first 25 years. 53 years later, the well-attended Circus continues to meet every spring and fall.

Contributions to FEM Theory and Practice¹

In the 1970s, Ivo's work centered on the development of a mathematical theory of FEM. One of his first American papers presented a formal framework for the rigorous numerical analysis of FEM. The paper contained one of Ivo's most important contributions: his introduction of the *inf-sup condition* and *discrete inf-sup condition* into numerical PDEs and his proof of their precise relation to the stability and convergence of Galerkin methods. In other publications, Ivo identified what became some of the major extensions of basic FEM, including *Lagrange multipliers*, *penalties* to enforce conformity or boundary conditions, and *treatment of singularities* in the data. Each of these topics later attracted scores of researchers — evidence of Ivo's prescience. In 1972, Ivo delivered an extraordinary series of 10 one-hour lectures during an international symposium on the mathematical foundations of FEM in Baltimore, Md. A written version of more than 350 pages was published in the meeting's proceedings and became the bible for the mathematical theory of finite elements. Later that decade, he introduced the famous *angle conditions*, wrote a seminal sequence of papers on *numerical homogenization*, and developed groundbreaking algorithms and theory for *a posteriori error estimation* and *mesh adaptivity*.

The 1980s were the years of the *p-version of FEM* wherein Ivo and his colleagues changed the asymptotical framework to increase polynomial degree instead of decreasing element size. Later in the decade, they developed the hybrid *hp-version of FEM*. Around the same time, Ivo also introduced *mesh-dependent norms*—a novel tool for the analysis of certain kinds of finite elements—and *generalized finite elements*, a new class of FEM.

Ivo entered the 1990s by coauthoring an authoritative survey article on *FEM for eigenvalue problems*. He then explored questions of *robustness* for different FEMs and played a big part in explicating *locking* behavior (the failure of standard numerical methods in certain parameter domains) and designing new methods to overcome it. Ivo conducted similar path-breaking work on *pollution*, another robustness issue that prevents accurate numerical solutions to the Helmholtz wave equation in the range of *large wave numbers*. Finally, some of his most cited work in the late 1990s details his invention of the *partition of unity finite element method* and study of other forms of *meshless methods*.

By this time, Ivo's work was widely respected and appreciated; he received three honorary doctorates (two more would come later), the Gold Medal of Charles University, the Birkhoff Prize in Applied Mathematics from the American Mathematical Society (AMS) and SIAM, and the John von Neumann Medal from the U.S Association for Computational Mathematics.

Texas: The Grand Old Man of FEM

In 1995, Ivo retired from UMD as an emeritus distinguished professor. However, the word "retire" is rather misleading. Ivo's longtime friend and collaborator, J. Tinsley Oden, recruited him to the University of Texas (UT) at Austin's Texas Institute for Computational and Applied Mathematics (TICAM)—now called the Oden Institute for Computational Engineering and Sciences—and Ivo moved to Texas. In addition to his primary role as TICAM Senior Research Fellow, he served as a professor in both the Department of Mathematics and Department of Aerospace Engineering and Engineering Mechanics, and was appointed Robert B. Trull Chair in Engineering. In 2016, the University named then-90-year-old Ivo as the first recipient of the W.A. "Tex" Moncrief, Jr. Distinguished Faculty Fellowship in Computational Sciences. But despite these many titles, a more descriptive term for Ivo's role in Texas is the Grand Old Man of Finite Elements. He continued to work on the topics that had occupied his life and also pursued some new directions, including stochastic PDEs and uncertainty quantification in finite element computations.

While at TICAM, Ivo was elected to the European Academy of Sciences and the U.S. National Academy of Engineering. He also received the Gauss-Newton Medal from the International Association for Computational Mechanics and the AMS Leroy P. Steele Prize for Lifetime Achievement. Ivo ultimately spent 23 years at UT Austin before officially retiring (again!) in 2018 at the age of 92.

Although Ivo had no formal teaching responsibilities at UT Austin, he remained very committed to training the next generation of computational scientists. Soon after his arrival, he founded the TICAM Forum (later renamed the Babuška Forum): a seminar series for students in contemporary computational science and engineering that is still active today.

In 2020, Renata Babuška—Ivo's wife of 63 years and his greatest collaborator—passed away. Ivo then moved to Albuquerque, NM, to be close to his son Vit. Ivo and Renata are also survived by their daughter Lenka and several grandchildren.

Ivo was exceptionally generous with his time and knowledge and always eager to talk about the subjects he loved. Later in life, he gave back to the community through philanthropy as well. In fact, a very generous donation by Ivo last year allowed SIAM to establish the [Ivo & Renata Babuška Prize](#): a biennial award in the amount of \$10,000 that recognizes "high-quality interdisciplinary work that targets any aspect of modeling and numerical solution of a specific engineering or scientific application, including mathematical modeling, numerical analysis, algorithms, and validation." The first iteration of the Ivo & Renata Babuška Prize will be presented at the 2025 SIAM Conference on Computational Science and Engineering. Ivo was equally charitable with other institutions that were closely connected to his career, including UMD, UT Austin, AMS, and several Czech scientific societies.

Ivo was a towering figure in a broad interdisciplinary field that combines mathematics, engineering, and computer science. He helped to shape and define the area that we now call computational science and engineering. He will be greatly missed by the numerous scientific communities that felt the impact of his work, and even more by his many friends and admirers. At the same time, let us celebrate the incredible legacy that he left behind.

¹Due to space limitations, this section only touches upon some of Ivo's most enduring contributions. His many collaborators are not mentioned by name.

Douglas N. Arnold is McKnight Presidential Professor of Mathematics at the University of Minnesota. He was a colleague of Ivo Babuška at the University of Maryland from 1979 to 1989, and a lifelong admirer and friend.