One question on the exam will be one of the following 6 theorems from Chapter 1 of H-H:

1. Linear transformations correspond to matrices (Theorem 1.3.4)

2. $\lim_{\underline{x} \to \underline{x}_0} f(\underline{x}) = \underline{a}$ iff for every sequence $\{\underline{x}_n\} \to \underline{x}_0$ $\lim_{n \to \infty} f(\underline{x}_n) = \underline{a}$

   (Theorem 1.5.27)

3. Bolzano - Weierstrass Thm (Theorem 1.6.3)

4. Mean Value Thm (assuming that every continuous function on $[a,b]$ attains a max/min)

   (Theorem 1.6.12)

5. The Jacobian is the derivative, if the derivative exists (Theorem 1.7.9)

6. If $f \in C^1(U)$, then f is differentiable on U (Theorem 1.9.8)

Proposition: Given a monomial $x_1^{a_1} \cdots x_n^{a_n}$ with $a_1 + \cdots + a_n = d$,

then
$$\lim_{\underline{x} \to \underline{0}} \frac{x_1^{a_1} \cdots x_n^{a_n}}{|x|^k} = \begin{cases} 0 & \text{if } d > k \\ \text{does not exist} & \text{if } d < k \end{cases}$$

i.e. not equal to a finite real #.

pf: if $d > k$, then we show $\lim_{\underline{x} \to \underline{0}} \frac{|x_1^{a_1} \cdots x_n^{a_n}|}{|x|^k} = 0$

which implies the desired result.

But $|x_i| \le |\underline{x}| = \sqrt{x_1^2 + \cdots + x_i^2 + \cdots x_n^2}$ so since $|x_1^{a_1} \cdots x_n^{a_n}| \le |x_1|^{a_1} \cdots |x_n|^{a_n}$

$$\lim_{\underline{x} \to \underline{0}} \frac{|x_1^{a_1} \cdots x_n^{a_n}|}{|\underline{x}|^k} \le \lim_{\underline{x} \to \underline{0}} \frac{|\underline{x}|^d}{|\underline{x}|^k}$$

$$= \lim_{\underline{x} \to \underline{0}} |\underline{x}|^{d-k} = 0.$$

If $d < k$, we can approach along the line $x_1 = x_2 = \cdots = x_n$, and ask whether $\lim_{x \to 0} \frac{x^d}{x^k}$ exists. if $d < k$, then this equals $\lim_{x \to 0} \frac{1}{x^{k-d}}$ which grows without bound as $x \to 0$.

Note: For polynomials, same proof shows the limit $\lim_{\underline{x} \to \underline{0}} \frac{p(\underline{x})}{|\underline{x}|^k} = 0$ if all monomials have degree $> k$ and DOES NOT EXIST if one monomial has degree $< k$