Name: ____________________

Problem Set 13
Math 4281, Fall 2013
Due: Monday, December 9

Read Sections 14.1, 14.2, as well as Theorem 9.6 in your textbook. Also, skim Sections 21.2, 23.1, 23.2, and 23.3. As the approach in the book is more detailed than in our notes, you need only pay attention to the definitions and examples. You are only responsible for your notes on these sections, not the book.

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1. Let D_4 act on itself by conjugation. How many orbits are there? What are the orbits?
2. If a group G of order 35 acts on a set S with 16 elements, show that the action must have a **fixed point**, i.e., there exists an $s \in S$ with $g \cdot s = s$ for all $g \in G$.
3. Let a group G act on itself by conjugation. Recall that the center of G is defined to be $Z = \{a \in G \mid ag = ga \text{ for all } g \in G\}$.
   a. Prove that $a \in Z \iff \mathcal{O}_a = \{a\} \iff G_a = G$.
   b. Prove that $Z \subseteq G_a$ for any $a \in G$ and that if $a \notin Z$, then $Z \subsetneq G_a \subsetneq G$.
4. Let G be a finite group of order n.
   a. For each $a \in G$, let $L_a \colon G \to G$ be defined by $L_a(g) = ag$. Prove that L_a is a permutation of G.
   b. Define $\phi \colon G \to \mathrm{Perm}(G)$ by $\phi(a) = L_a$. Prove that ϕ is a one-to-one group homomorphism.
   c. Use (b) to prove that G is isomorphic to a subgroup of S_n.
5. Determine the Galois group and the corresponding subgroups and intermediate fields for $f(x) = x^3 + 2 \in \mathbb{Q}[x]$. Check for normal subgroups and Galois extensions of $\mathbb{Q}$.

Through the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ____________________