Name: ____________________

Problem Set 3
Math 4281, Fall 2013
Due: Friday, September 27

Read Sections 16.1, 16.2, and 4.2 in your textbook. (In Section 16.2, we will only use a special case of reading from Theorem 16.4 to Theorem 16.6, as abbreviated in class notes. You will not be responsible for the more general results stated in this portion of the text.)

Complete the following items, staple this page to the front of your work, and turn your assignment in at the beginning of class on Friday, September 27.

1. Apply the division algorithm to the polynomials $f(x), g(x) \in \mathbb{Z}_7[x]$, where
$$f(x) = x^6 + \overline{3}x^5 + \overline{4}x^2 - \overline{3}x + \overline{2} \quad \text{and} \quad g(x) = x^2 + \overline{2}x - \overline{3}.$$
Clearly identify $q(x)$ and $r(x)$.

2. Find the greatest common divisor $d(x)$ for the polynomials $f(x), g(x) \in \mathbb{C}[x]$, where
$$f(x) = x^2 + 1 \quad \text{and} \quad g(x) = x^2 - i + 2,$$
and find $s(x), t(x) \in \mathbb{C}[x]$ to express $d(x) = s(x)f(x) + t(x)g(x)$.

3. Show that unique factorization fails in $R[x]$ when R is not an integral domain. Consider, for example, $x^2 + x + \overline{8} \in \mathbb{Z}_{10}[x]$.

4. Decide whether or not the following polynomials are irreducible.
   a. $x^2 + \overline{1} \in \mathbb{Z}_5[x]$
   b. $x^2 + \overline{1} \in \mathbb{Z}_{19}[x]$
   c. $x^3 - \overline{9} \in \mathbb{Z}_{11}[x]$

5. Find all odd prime integers p so that $x + \overline{2}$ is a factor of $f(x) = x^4 + x^3 + x^2 - x + \overline{1} \in \mathbb{Z}_p[x]$.

6. Complete the following exercises in your textbook.
   pp. 284-5 # 9, 26

Through the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ____________________