Name: ______________________________

Problem Set 9

Math 4281, Fall 2013

Due: Friday, November 8

Read Sections 3.1 and 3.2 in your textbook.

---

1. Which of the following are groups?

   a. $\{1, 3, 7, 9\} \subseteq \mathbb{Z}_{10}$, with operation multiplication
   b. $\{0, 2, 4, 6\} \subseteq \mathbb{Z}_{10}$, with operation addition
   c. $\{x \in \mathbb{Q} \mid 0 < x \leq 1\}$, with operation multiplication
   d. the set of all positive irrational real numbers, with operation multiplication
   e. the set of imaginary numbers $\{ix \mid x \in \mathbb{R}\}$, with operation addition
   f. $\{z \in \mathbb{C} \mid |z| = 1\}$, with operation multiplication
   g. $\mathbb{Z}$ with operation $a \bullet b = a + b + 1$
   h. $\mathbb{Z}$ with operation $a \bullet b = a - b$
   i. $\mathbb{Q} \setminus \{1\}$ with operation $a \bullet b = a + b - ab$

2. Prove or give a counterexample. If G is a group and $a, b, c \in G$ with $ab = bc$, then $a = c$.

3. Let G be a group with identity element e.

   a. Prove that $(ab)^2 = a^2b^2$ for all $a, b \in G$ if and only if G is abelian.
   b. Prove that if every element $a \in G$ is such that $a^2 = e$, then G is abelian.

4. Let G be a group and fix $a \in G$. Prove that $C_a = \{x \in G \mid ax = xa\}$ is itself a group, called the centralizer of a.

Through the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ______________________________