Name: ______________________

Problem Set 2
Math 4281, Spring 2014
Due: Wednesday, February 5

Complete the following items, staple this page to the front of your work, and turn your assignment in class on Wednesday, February 5.

Properties of the integers

1. Prove that the square of an even number is even and the square of an odd number is odd.

Division and Euclidean algorithms

2. Using the division algorithm, show that every perfect square (i.e., a number of the form n^2) is of the form $4k$ or $4k+1$ for some nonnegative integer k.
3. For the pairs of numbers a and b, calculate $\gcd(a,b)$ and find integers r and s such that $\gcd(a,b) = ra + sb$.
   (a) 234 and 165
   (b) 1739 and 9923
   (c) 23771 and 19945
4. Define the *least common multiple* of two nonzero integers a and b, denoted by $\operatorname{lcm}(a,b)$, to be the nonnegative integer m such that both a and b divide m, and if a and b divide any other integer n, then m also divides n. Prove that any two nonzero integers a and b have a unique least common multiple.
5. If $d = \gcd(a,b)$ and $m = \operatorname{lcm}(a,b)$, prove that $dm = |ab|$.
6. Using the fact that 2 is prime, show that there do not exist integers p and q such that $p^2 = 2q^2$. Demonstrate that therefore $\sqrt{2}$ cannot be a rational number.

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ______________________