Name: ______________________

Problem Set 3
Math 4281, Spring 2014
Due: Wednesday, February 12

Complete the following items, staple this page to the front of your work, and turn your assignment in class on Wednesday, February 12.

Division and Euclidean algorithms

1. You have at your disposal arbitrarily many 4-cent stamps and 7-cent stamps. What are the postages you can pay? Show in particular that you can pay all postages greater than 17 cents.

Modular arithmetic

2. Suppose p is prime and a and b are integers. Prove that if $a^2 \equiv b^2 \pmod{p}$, then $a \equiv b \pmod{p}$ or $a \equiv -b \pmod{p}$.
3. Determine the last digit of 3^{400}, then the last two digits. Determine the last digit of 7^{99}.
4. Prove that there are infinitely many primes of the form $4n - 1$.

Solving congruences

5. Prove that if $x^2 \equiv n \pmod{65}$ has a solution, then so does $x^2 \equiv -n \pmod{65}$.
6. Solve the following congruences:
   a. $6x + 3 \equiv 1 \pmod{10}$
   b. $15x \equiv 25 \pmod{35}$
   c. Simultaneously: $x \equiv 1 \pmod{4}$, $x \equiv 7 \pmod{13}$
   d. Simultaneously: $x \equiv 11 \pmod{142}$, $x \equiv 25 \pmod{86}$

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ______________________