

MATH 8211 PAPERS - FALL 2017

This project should tackle a subject in commutative algebra that you are interested in learning about. Your end product will be a short paper (3-5 pages, typed) summarizing some of the main results about this subject. It should include at least one theorem statement (with proof only if appropriate for a paper of this length), as well as an example illustrating the main concept.

To begin, choose a topic and a reasonable reference for it. Send this info to me by email by the deadline below. Topics may not be repeated, and they are available on a first-come, first-serve basis.

I am happy to help you design or refine your project as much as needed. Your project grade will be based on your paper's content and clarity, as well as a peer review of a classmate's rough draft.

References. Roughly speaking, you could choose to study about 10-15 pages in any of the following books, where you should be able to find a good topic:

- [BH]: Bruns and Herzog's *Cohen–Macaulay Rings*
- [CLO2]: Cox, Little, and O'Shea's *Using Algebraic Geometry*
- [E]: Eisenbud's *Commutative Algebra with a view towards algebraic geometry*
- [GoS]: Eisenbud's *The Geometry of Syzygies*
- [MS]: Miller and Sturmfels's *Combinatorial Commutative Algebra*
- [P]: Peeva's *Graded Syzygies*
- Any other book or paper that you think would be relevant.

Here are some more specific ideas. It is important to note that these topics generally contain far more material than would be appropriate for your project. That means that your job is to identify the results that you think are most interesting and then present them in an accessible and condensed manner. This is much harder than it may sound at first!

- (1) Symbolic powers [E, 3.9]
- (2) Filtrations and the Artin-Rees Lemma [E, 5]
- (3) Blow-up algebras [E, 5.2, 6.5]
- (4) Local criterion for flatness [E, 6.4]
- (5) What makes a Complex Exact? [E, 20.3]
Eisenbud's remark on his famous theorem: "In a sense, [this theorem] shows that the condition for a complex to be exact is not so complicated; but being a complex is a very subtle property!"
- (6) The Hilbert–Burch Theorem [E, 20.4] and possibly some from [GoS]
This is one of the original theorems in homological commutative algebra. It has had many applications to projective geometry.
- (7) Castelnuovo–Mumford Regularity [E, 20.5] or [GoS, 4].
This invariant "controls" the complexity of ideals in a certain sense. If you're interested in algebraic geometry, you might also look at the remarkable paper "What can be computed in algebraic geometry" by Bayer and Mumford.
- (8) Canonical modules and Gorenstein rings [E, 21] or [BH, 3]
This is such an essential topic in modern commutative algebra, but we won't have time to get to it! The three references are so different that multiple groups could each take this topic.

- (9) Stanley–Reisner rings [BH, II.5] or [MS, I.1] or [P, 61-62]
This theory provides a dictionary between simplicial complexes and squarefree monomial ideals, and it was the key breakthrough in Richard Stanley’s proof of the Upper Bound Conjecture.
- (10) Solving polynomial equations [CLO2, 2]
How do you actually find actually the solutions to a system of polynomial?
- (11) Resultants [CLO2, 3]
- (12) Bernstein’s Theorem for solving systems of sparse equations [CLO2, 7.4-7.5].
This is a really cool variant of Bezout’s theorem. I think the proof is pretty serious, but even the statement and applications are pretty interesting.
- (13) Algebraic coding theory [CLO2, 9]
- (14) Alexander duality [MS, I.5]
- (15) Semigroup rings [MS, II.7]
This is closely related to the study of toric varieties.
- (16) Multigraded polynomial rings [MS, II.8]
This is also closely related to the study of toric varieties.
- (17) Macaulay’s Theorem on Hilbert functions [BH, 4.1–4.2]
It takes a bit of work to unravel the notation, but this is a spectacular theorem.
- (18) Field theory: transcendence degree [E, A1]
- (19) Geometry from syzygies [GoS, 2C]
This section refers to other results in the book, which you’ll also need.
- (20) Determinantal rings [BH, 7]
- (21) Plücker coordinates [MS, III.14]
- (22) Tate resolution and infinite free resolutions [P, 32,33]
Tate showed that over a local ring, the resolution of the residue field can be infinite, but it has some very interesting structure.
- (23) Numerical algebraic geometry
Bertini is a program that uses numerical techniques, as opposed to Gröbner bases, to perform algebraic computations. Learn about the ideas underlying the program. This project is probably more advanced, since it will require more independent work.
- (24) Make your own project!

Important dates.

- September 18: deadline for topic choice, submitted to me by email
- October 2: 1 hard copy of typed outline for paper due (A paragraph or two that clearly outlines the project and that includes some possible references.)
- October 23: 1 hard copy of typed progress report due (A 1-2 page writeup that demonstrates progress on both the expository and computational sides of the project.)
- November 13: 2 hard copies of typed rough draft due in class for peer review (This should be a fairly complete rough draft.)
- November 20: 2 hard copies of typed peer review report due in class (This should be a 1-2 paragraph report that summarizes the main point of the paper under review and weighs in on the strengths and weaknesses of the write-up.)
- December 6: 1 hard copy of project paper due in class and another copy through email by noon