## HOMEWORK \#3 (DUE WEDNESDAY, MARCH. 25).

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03 / 12 / 2015
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Note: Turn in only the "starred" problems; out of these, selected problems will be graded.

Section 12.1: Exercises 2, 6, $9^{*}$, 17, 18, 19, 21*.
Section 12.2: Exercises 6, 8, 18*, 19.
Section 12.3: Exercises 1, 2, 17*, 25, 26*, 31, 32*, 33, 34*.

## Additional problems:

1. Prove that if $N$ is a nilpotent matrix, then $I+N$ is invertible.
$\mathbf{2}^{*}$. Let $E$ be a finite dimensional vector space over a field $k$ and let $A: E \longrightarrow E$ be a $k$-linear endomorphism. We say that $A$ is diagonalizable if there exists a basis of $E$ consisting of eigenvectors of $A$. Prove that $A$ is diagonalizable if and only if the minimal polynomial $q_{A}(t)$ has the form

$$
q_{A}(t)=\prod_{i=1}^{d}\left(t-\lambda_{i}\right)
$$

with $\lambda_{1}, \ldots, \lambda_{d}$ distinct elements of $k$.

