Homework #2: Krylov subspaces methods. Due on Monday, March 4.

In what follows, we apply some Krylov subspaces methods to numerically invert the matrix equation

$$A\,x = b$$

associated to the five-point finite difference approximation of the model problem

$$-\Delta u = f \quad \text{in } \Omega := (0,1)^2, \qquad u = 0 \quad \text{on } \partial\Omega.$$

We use uniform Cartesian grids of squares of size $h = 1/N$.

1. (8 points) The objective of this exercise is to compare the performance of the steepest descent (SD) and the method of conjugate gradients (CG) for $f = 1$ and the initial guess $x_0 = 0$. For each method and the mesh associated to $h = 1/N$, compute the number of iterations needed to reduce the **energy** norm of the initial error 10^6 times. Do this for $N = 2, 4, 8, ...$ Do your results agree with the theory?

Table 0.2

Number of iterations to reduce the initial error a million times

N	SD	CG
2		
4		
8		
...		

2. (4 points) Compare the number of operations of the CG and the GMRES methods. Consider that an addition and a multiplication is one "operation". Also compare the memory requirements of the two methods.

3. (8 points) The objective of this exercise is to explore the performance of the GMRES method for $f = 1$ and the initial guess $x_0 = 0$ in terms of the maximum dimension of its Krylov subspace, m. (We re-start the GMRES method each m iterations and take the initial guess to be the last iterate.) For each method and the mesh associated to $h = 1/N$, compute the number of iterations needed to reduce the **energy** norm of the initial error 10^6 times. Do this for $N = 2, 4, 8, ...$ and several values of m (se an example below). How does the value of m affect the convergence? Is there any way to pick the optimal value? Do your results agree with the theory?

TABLE 0.3
Number of iterations to reduce the initial residual a million times

N	$m = N$	$m = [\sqrt{N}]$	$m = 20$
2			
4			
8			
$\ldots$			