ANALYSIS of NUMERICAL METHODS for PDEs
MATH 8445-6 - FALL 2019/SPRING 2020, VH 301 10:10-11:05 am
Instructor: B. Cockburn

- **Objective**: This course is intended to be an introduction to the *mathematical analysis* of finite element methods for partial differential equations. The distinctive feature of this course is that it emphasizes the interplay between theory and practice, as well as the physics behind the devising of the different numerical methods.

- **References**:
  - D. Braess, Finite Elements. Theory, Fast Solvers and Applications in Solid Mechanics;
  - F. Brezzi and M. Fortin, Mixed and hybrid finite element methods;
  - P.G. Ciarlet, The Finite Element Method for Elliptic Problems;
  - C. Johnson, Numerical solutions of PDEs by the Finite Element Method;
  - R. LeVeque, Numerical methods for conservation laws;
  - R. LeVeque, Finite Volume Methods for Hyperbolic Problems;
  - A. Quarteroni and A. Valli, Numerical methods for PDEs.

•**Grade**: Average of homeworks. (One each two weeks)

•**Office hours**: By appointment (cockburn@math.umn.edu).

•**Program for the Fall**:

- Linear hyperbolic conservation laws (R. LeVeque)
  - The linear scalar hyperbolic conservation law.
  - The Discontinuous Galerkin method.
  - Upwinding as artificial diffusion.
  - Error estimates.
  - The Runge-Kutta discontinuous Galerkin method.
  - The wave equation, Maxwell's equations, Friedrich's systems.
  - The Riemann problem and the artificial viscosity matrix.
- Non-linear conservation laws (R. LeVeque + Notes)
  - Weak solutions.
  - The entropy solution.
  - The Riemann problem.
  - Monotone schemes.
  - The Runge-Kutta discontinuous Galerkin method.
  - The slope limiter as artificial viscosity.
  - The Euler equations of gas dynamics.
- The continuous Galerkin method for the Poisson equation. (Johnson)
  - Variational formulation.
  - A geometric interpretation.
  - A minimization problem.
  - Static condensation.
  - The Neumann problem.
  - Examples of finite elements.

  - Error analysis.
  - Abstract formulation and error analysis.
  - Eigenvalue problems
- Approximation theory for finite element methods. (Johnson, Ciarlet)
  - Interpolation with linear functions.
  - Interpolation with higher degree polynomials.
  - Error estimates.
  - Regularity of the exact solution.
  - Error estimates in the L^2-norm.

•**Program for the Spring**:

- Mixed and HDG methods for the heat equation. (Notes)
  - Static condensation.
  - Devising the HDG methods.
  - Existence and uniqueness.
  - Implementation.
  - Minimization problem.
  - The saddle point structure
  - Error analysis.
- Convection-diffusion problems (Johnson)
  - The model convection-diffusion problem
  - Continuous Galerkin methods.
  - Artificial diffusion.
  - Streamline-diffusion methods.
  - Discontinuous Galerkin methods.
- Mixed and HDG methods for Fluid flow (Johnson, Brezzi and Fortin)
  - The Stokes system of fluid flow.
  - Different approaches to incompressibility
  - The Oseen's equations.
  - The incompressible Navier-Stokes equations.
  - The compressible Navier-Stokes equations.
- Mixed and HDG methods for Elasticity (notes)
  - The equations of linear elasticity.
  - Methods with weakly symmetric stresses.
  - Methods with strongly symmetric stresses.
  - Nonlinear elasticity.
  - Wave propagation in elastic media.