ANALYSIS of NUMERICAL METHODS for PDEs
MATH 8445-6 - FALL 2020/SPRING 2021
MWF 11:15-12:05, Zoom ID: 943 8131 0813
Instructor: B. Cockburn

- **Objective**: This course is intended to be an introduction to the *mathematical analysis* of finite element methods for partial differential equations. The distinctive feature of this course is that it emphasizes the interplay between theory and practice, as well as the physics behind the devising of the different numerical methods.

- **References**:
  - D. Braess, Finite Elements. Theory, Fast Solvers and Applications in Solid Mechanics;
  - F. Brezzi and M. Fortin, Mixed and hybrid finite element methods;
  - P.G. Ciarlet, The Finite Element Method for Elliptic Problems;
  - C. Johnson, Numerical solutions of PDEs by the Finite Element Method;
  - R. LeVeque, Numerical methods for conservation laws;
  - R. LeVeque, Finite Volume Methods for Hyperbolic Problems;
  - A. Quarteroni and A. Valli, Numerical methods for PDEs.

•**Grade**: Average of homeworks. (One per month)

•**Office hours**: By appointment (cockburn@math.umn.edu).

•**Program for the Fall**:

- Review of approximation by (piecewise) polynomials. (Notes)
  - Interpolation with linear functions.
  - Interpolation with higher degree polynomials.
  - Error estimates.
  - Regularity of the exact solution.

- Linear hyperbolic conservation laws (R. LeVeque + Notes)
  - The linear scalar hyperbolic conservation law.
  - The Discontinuous Galerkin method.
  - Upwinding as artificial diffusion.
  - Error estimates.
  - The Runge-Kutta discontinuous Galerkin method.
  - Systems
  - Acoustic, elastic and electromagnetic waves.
  - The Riemann problem and the artificial viscosity matrix.
  - General Friedrich's systems.
  - Hamiltonian systems and symplectic time-marching methods.

- Non-linear conservation laws (R. LeVeque + Notes)
  - Weak solutions.
  - The entropy solution.
  - The Riemann problem.

- Monotone schemes.
- High-resolution schemes.
- The Runge-Kutta discontinuous Galerkin method.
- The slope limiter as artificial viscosity.
- Nonlinear elastic large deformations.
- The Euler equations of gas dynamics.

•**Program for the Spring**:

- Finite element methods methods for diffusion. (Notes, Johnson, Ciarlet)
  - The methods (Continuos, Mixed and Discontinuos Galerkin).
  - Existence and uniqueness.
  - Static condensation.
  - Implementation.
  - The minimization problem.
  - The saddle point structure
  - Error analysis.

- Methods for Elasticity (Notes)
  - The equations of linear elasticity.
  - Methods with weakly symmetric stresses.
  - Methods with strongly symmetric stresses.
  - Nonlinear elasticity.

- Convection-diffusion problems (Johnson)
  - The model convection-diffusion problem
  - Continuous Galerkin methods.
  - Streamline-diffusion methods.
  - Discontinuous Galerkin methods.

- Methods for Fluid flow (Johnson, Brezzi and Fortin)
  - The Stokes system of fluid flow.
  - New approaches to incompressibility
  - The incompressible Navier-Stokes equations.
  - The compressible Navier-Stokes equations.