Topics in Numerical Analysis
MATH 8450

Instructor: Bernardo Cockburn. Office: Vincent Hall 327

Fall 2019 schedule: MWF 1:25-2:15, Vincent Hall 2

- **Objective**: In this course we study the so-called hybridizable discontinuous Galerkin (HDG) methods. These methods can be applied to a wide variety of partial differential equations, can handle unstructured meshes, adaptivity, provide high-order accurate approximations and have a built-in stabilization mechanism that renders them very robust. We are going to systematically study how to devise the methods as they are applied to problems appearing in fluid dynamics and solid mechanics. Some knowledge of finite element methods is welcome.
- **Notes**: Reading material will be provided by the instructor.
- **Program**: We are going to consider the following topics:
  (1) The original DG method for linear transport.
  (2) The HDG methods for steady-state diffusion.
  (3) For steady-state convection-diffusion.
  (4) For time-dependent convection-diffusion.
  (5) For the acoustic wave equation.
  (6) For the Stokes system of incompressible flow.
  (7) For the incompressible Navier-Stokes equations.
  (8) For linear elasticity.
  (9) For large deformation elasticity.
  (10) For elastodynamics.
  (11) For time-harmonic Maxwell equations
  (12) For electromagnetic wave propagation.

The plan is to cover chapters (1), (2), (5), (6) in full detail, because they develop the basic components of the devising of the HDG methods. Chapter (1) describes the original DG method and is devoted to showing that this method (devised back in 1973) already contains all the main features of the HDG methods. Chapter (2) is the main material of the course. It develops how to devise the HDG methods (introduced in 2009), why they are well defined, how to implement them, the different ways of presenting the methods (including in terms of new variational formulations), and the relation to already known, well established methods for steady-state diffusion. Chapter (5) discusses how to define HDG methods for wave propagation problems. It revisits the so-called upwinding flux as the choice that can provide superconvergent approximations. It show how to take into account the different structures of the acoustic wave equation, namely, the structure of a Friedrichs system and the Hamiltonian structure, to define different types of numerical schemes. Finally, chapter (6) deals with the treatment of the incompressibility condition on the velocity. Other chapters will be considered depending on the interest of the class.

- **Work**: Feedback about the reading material distributed in class will be most welcome. Coding and testing the methods is usually a good idea to understand them, but in this course it is only optional.
- **Office hours**: By appointment only. E-mail me to cockburn@math.umn.edu.