Quiz 7

Math 1572H, 23 March 2006

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1. [4 points] Write the Taylor polynomial of degree 3 for $f(x) = \tan(x)$ about $x = 0$.

Solution: We'll need some ingredients to plug into the formula. First off,

$$\begin{aligned} f'(x) &= \sec^2 x \\ f''(x) &= 2\sec x(\sec x \tan x) = 2\sec^2 x \tan x \\ f^{(3)}(x) &= 2\sec^4 x + 4\sec^2 x \tan^2 x \end{aligned}$$

Using this, we find that $f(0) = f''(0) = 0$, $f'(0) = 1$, and $f^{(3)}(0) = 2$. Plugging this into the formula for Taylor polynomials gives

$$T_3(x) = x + \frac{x^3}{6}.$$

2. [6 points] In terms of x, find the sum of the power series

$$x + 2x^2 + 3x^3 + \cdots + nx^n + \cdots .$$

What is the interval of convergence for this series?

Solution:

$$\begin{aligned} x + 2x^2 + 3x^3 + \ldots &= \sum_{n=0}^{\infty} nx^n \\ &= x\sum_{n=0}^{\infty} nx^{n-1} \\ &= x\sum_{n=0}^{\infty} \frac{d}{dx}\left[x^n\right] \\ &= x\frac{d}{dx}\left[\sum_{n=0}^{\infty} x^n\right] \\ &= x\frac{d}{dx}\left[\frac{1}{1-x}\right] \\ &= x\left(\frac{1}{(1-x)^2}\right). \end{aligned}$$

Note that in the second line, we tacitly assumed that $x \neq 0$. However, the formula we came up with for the sum of the series clearly holds even if $x = 0$.

The radius of convergence is 1, as can be seen by the ratio test, the formula given in the book for R, or by noting the fact that the radius of convergence for a series and its derivative are the same (and knowing the radius of convergence for the geometric series). Neither the endpoint $x = 1$ nor $x = -1$ will give convergent series, because the terms don't go to zero. Therefore, the interval of convergence is $(-1, 1)$.

3. (a.) [0 points] What is Ryan's favorite number? 17
(b.) [1 point] Why is this Ryan's favorite number?

Solution: See

`http://brewers.mlb.com/NASApp/mlb/mil/ballpark/walk_of_fame.jsp`

class of 2004. Uniform number 17, 17 seasons, almost 1700 hits, and a beer garden named after him. Yet unknown in Minnesota, eh?