Worksheet 5

Math 1572H, 21 March 2006

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1. Taylor polynomials Find the Taylor polynomial of degree 3 about $x = 0$ for the function $f(x) = \ln(\sin(x + \pi/2))$.

Answer: $T_3(x) = 1 - x^2/2$.

2. Booyah Find the Taylor polynomial of degree 3 about $x = 0$ for the function $g(x) = \sqrt{1+x}$.

Answer: $T_3(x) = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$.

3. Expected value and variance of a random variable The expected value of a (discrete) random variable X is given by the formula

$$E[X] = \sum_n nP(X = n).$$

The variance of a random variable X is given by the formula

$$V[X] = E[X^2] - (E[X])^2.$$

Typically, one uses the standard deviation (the square root of the variance) to measure the spread of a random variable.

3a. If X is a geometric random variable with parameter q, then

$$P(X = n) = \begin{cases} q^{n-1}(1-q) & \text{for } n \geq 1 \\ 0 & \text{for } n \leq 0. \end{cases}$$

Find the expected value and the variance of a geometric random variable with parameter q.

Answer: The expected value is $1/(1-q)$ and the variance is $q/(1-q)^2$.

3b. If Y is a Poisson random variable with parameter λ, then

$$P(Y = n) = \begin{cases} \frac{\lambda^n e^{-\lambda}}{n!} & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

Find the expected value and variance of a Poisson random variable with parameter λ.

Answer: The expected value and variance are both λ. Please let me know if you would like to see more details here. I would be happy to type up the solutions for you.