Solutions 1

#1 Find the greatest common divisor of x^2+x+1 and x^4+x^3+x+1 in $k[x]$ where $k = \mathbf{Z}/2$.

Use the Euclidean Algorithm for polynomials over a field: first, reduce x^4+x^3+x+1 mod x^2+1 by dividing-with-remainder:

The Euclidean algorithm (with divisions done just below):

$$(x^4+x^3+x+1) \quad - \quad (x^2+1)(x^2+x+1) \quad = \quad 0$$

Since we got a 0, the divisor itself is the gcd: it is x^2+x+1.

$$\begin{array}{r|l}
 & x^2 \ +0 \ +1 \text{ R } 0 \\ \hline
x^2 \ +x \ +1 & x^4 \ +x^3 +0 \ +x \ +1 \\
 & x^4 \ +x^3 +x^2 \\ \hline
 & \qquad\ \ x^2 \ +x \ +1 \\
 & \qquad\ \ x^2 \ +x \ +1 \\ \hline
 & \qquad\qquad\quad 0
\end{array}$$

#2 Find the greatest common divisor of x^2+x+1 and x^4+x^3+x+1 in $k[x]$ where $k = \mathbf{Z}/3$.

Use the Euclidean Algorithm. Divisions are shown after.

$$\begin{array}{ccccc}
(x^4+x^3+x+1) & - & (x^2-1)(x^2+x+1) & = & 2x+2 \\
(x^2+x+1) & - & (2x)(2x+2) & = & 1
\end{array}$$

Since $1 \in k^\times$, the gcd is 1.

$$\begin{array}{r|l}
 & x^2 \ +0 \ -1 \text{ R } 2x+2 \\ \hline
x^2 \ +x \ +1 & x^4 \ +x^3 +0 \ +x \ +1 \\
 & x^4 \ +x^3 +x^2 \\ \hline
 & \qquad -x^2 +x \ +1 \\
 & \qquad -x^2 -x \ -1 \\ \hline
 & \qquad\qquad 2x \ +2
\end{array}$$

$$\begin{array}{r|l}
 & 2x \ +0 \text{ R } 1 \\ \hline
2x \ +2 & x^2 \ +x \ +1 \\
 & x^2 \ +x \\ \hline
 & \qquad\quad 1
\end{array}$$