Math 5615H: Introduction to Analysis I. Fall 2017

Homework #4, due Weds. October 4. 50 points.

#1. Let f be a mapping of A to B. Show that for each $B_1 \subseteq B$ and $B_2 \subseteq B$, their **inverse images** satisfy the properties

$$(i) \quad f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2), \qquad (ii) \quad f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2).$$

#2. Let f be a mapping of A to B. Verify whether or not the **images** of subsets $A_1 \subseteq A$ and $A_2 \subseteq A$ in general satisfy the properties

$$(i) \quad f(A_1 \cup A_2) = f(A_1) \cup f(A_2), \qquad (ii) \quad f(A_1 \cap A_2) = f(A_1) \cap f(A_2).$$

#3. The **set difference** of A and B by definition is $A \setminus B := \{x \in A : \ x \notin B\}$. Simplify the expressions

$$(a) \quad A \setminus (B \setminus A), \qquad (b) \quad A \setminus (A \setminus B), \qquad (c) \quad A \cap (B \setminus A).$$

#4. (a) Suppose that $E \subset \mathbf{R^k}$ is a bounded set in $\mathbf{R^k}$, and that $\varepsilon > 0$. **Show that** there is a finite set of points $\{p_1, p_2, \ldots, p_n\}$ in $\overline{E}$ so that for any point $x \in E$ there is $1 \le i \le n$ so that $|x - p_i| < \varepsilon$. (b) For the set $E = (-2, 2) \times (-2, 2) \times (-2, 2) \subset \mathbf{R^3}$, find an explicit choice of $\{p_1, p_2, \ldots, p_n\}$ which works for any $\varepsilon > \frac{1}{\sqrt{3}}$.

#5. Show that the unit interval $I_1 := \{x : \ 0 \le x \le 1\}$ is equivalent to (is in one-to-one correspondence to) the unit square $I_2 := \{x = (x_1, x_2) : \ 0 \le x_1, x_2 \le 1\}$. *Hint.* Use decimal representation of $x \in I_1$.