

In joint work with Carlos E. Kenig, we prove a Carleman inequality of the form

$$\begin{aligned} \|e^{\beta\varphi_\lambda(x_1)}u\|_{X'} &\leq C\|e^{\beta\varphi_\lambda(x_1)}(i\partial_t + \Delta_x)u\|_X \\ &\quad + C[\|e^{\beta\varphi_\lambda(x_1)}u(\cdot, -1)\|_{L^2(\mathbb{R}^n)} + \|e^{\beta\varphi_\lambda(x_1)}u(\cdot, 1)\|_{L^2(\mathbb{R}^n)}] \end{aligned}$$

for suitable functions  $u : \mathbb{R}^n \times [-1, 1] \rightarrow \mathbb{C}$ , any  $\beta \geq 0$ , and any  $\lambda \geq \Lambda(\beta)$ . The spaces  $X$  and  $X'$  are Strichartz spaces and the function  $\varphi_\lambda : \mathbb{R} \rightarrow \mathbb{R}$  is smooth,  $\varphi_\lambda(x_1) = x_1$  if  $x_1 \leq \lambda$  and  $\varphi_\lambda(x_1) = C_\lambda$  if  $x_1 \geq 2\lambda$ . We use this Carleman inequality to prove uniqueness of solutions of the nonlinear Schrödinger equation

$$(i\partial_t + \Delta_x)u = Vu + F(u), \tag{0.1}$$

where  $V$  is a potential and  $F$  is a nonlinear term. Under suitable assumptions on  $V$  and  $F$  we prove that if  $u_1$  and  $u_2$  are solutions to (0.1) in a suitable function space, and if  $u_1$  and  $u_2$  agree at times  $t = 0$  and  $t = 1$  in a half space, respectively in the complement of a convex cone, then they agree at all times in a half space, respectively in the entire  $\mathbb{R}^n$ .