

THE SEL'KOV MODEL: A SIMPLE PDE SYSTEM WITH COMPLEX BEHAVIOR

GARY M. LIEBERMAN

In 1968, E. E. Sel'kov proposed a simple kinetic model for enzyme catalysis which (after some simplifications) can be written as the pair of ODEs: $u' = 1 - uv^p$, $v' = \alpha(uv^p - v)$. For suitable values of the parameters α and p , this system has a limit cycle in addition to the unique steady state $u = v = 1$. This limit cycle corresponds to periodic oscillations in the concentration of the two chemicals which are catalyzed. Various authors have since studied the modification of this system which allows for diffusion: $-u_t + d_1 \Delta u = uv^p - 1$, $-v_t + \Delta v = \alpha(v - uv^p)$ with either Dirichlet or Neumann boundary conditions, primarily in one or two spatial dimensions. We shall give a brief survey of results for the reaction-diffusion system including applications beyond Sel'kov's and recent results for more than two spatial dimensions.