Boltzmann Diffusive Limit Beyond the Navier-Stokes Approximation

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Abstract

Given a normalized Maxwellian μ and $n \geq 1$, we establish the global in time validity of a diffusive expanison

$$F^{\varepsilon}(t,x,v) = \mu + \sqrt{\mu}\{\varepsilon f_1(t,x,v) + \varepsilon^2 f_2(t,x,v) + ...\varepsilon^n f_n^{\varepsilon}(t,x,v)\}, \quad (1)$$

for a solution F^{ε} to the rescaled Boltzmann equation (diffusive scaling)

$$\varepsilon \partial_t F^{\varepsilon} + v \cdot \nabla_x F^{\varepsilon} = \frac{1}{\varepsilon} Q(F^{\varepsilon}, F^{\varepsilon}), \quad (2)$$

inside a periodic box $\mathbf{T}^3$. We assume that in the initial expansion (1) at $t = 0$, the coefficients $f_m(0,x,v)$ has arbitary divergent free velocty field as well as tempertature field for all $1 \leq m \leq n$ while $f_1(0,x,v)$ has small amplitude in H^2. For $m \geq 2$, $f_m(t,x,v)$ are determined by a sequence of linear Navier-Stokes-Fourier systems iteratively. More importantly, the remainder $f_n^{\varepsilon}(t,x,v)$ is proven to decay in time uniformly in ε, via an unified nonlinear energy method. In particluar, our reuslts lead to error estimate for $f_1(t,x,v)$, the well-known Navier-Stokes-Fourier approximation and beyond. The collision kernel Q includes hard-sphere, the cutoff inverse-power, as well as the Coulomb interactions.