Doyoon Kim, School of Mathematics, University of Minnesota

Elliptic equations with L_∞ VMO coefficients

We study the L_p-theory of the elliptic differential equation

$$a^{jk}(x)u_{x^jx^k}(x) + b^j(x)u_{x^j}(x) - c(x)u(x) = f(x) \quad \text{in} \quad \mathbb{R}^d,$$

where $a^{jk}(x)$ are allowed to be only measurable with respect to one coordinate, say $x^1 \in \mathbb{R}$, where $x = (x^1, x') \in \mathbb{R}^d$, $x' \in \mathbb{R}^{d-1}$. Specifically, we show that there exists a unique solution to the above equation in $W_p^2(\mathbb{R}^d)$, $p \in (2, \infty)$, under the assumption that $a^{jk}(x^1, x')$ are measurable in $x^1 \in \mathbb{R}$ and VMO in $x' \in \mathbb{R}^{d-1}$. If the coefficients a^{jk} are independent of $x' \in \mathbb{R}^{d-1}$ (more generally, uniformly continuous in $x' \in \mathbb{R}^{d-1}$), then the equation is uniquely solvable in $W_p^2(\mathbb{R}^d)$, $p \in [2, \infty)$. In addition, we show that one can easily solve the equation in a half space, say $\mathbb{R}^d_+ = \{(x^1, x') : x^1 > 0, x' \in \mathbb{R}^{d-1}\}$, using the results for equations in the whole space.

This is a joint work with Nicolai Krylov.