NONVARIATIONAL METHODS FOR SEMILINEAR ELLIPTIC EQUATIONS OF CRITICAL GROWTH

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Nonlinear elliptic equations model a wide variety of physical applications. The particular type of equation in which I am interested takes the form

$$\Delta u - \vec{c} \cdot \nabla u = \lambda u + g(x, \lambda)|u|^{p-2}u \tag{0.1}$$

where $\vec{c}$ is a constant vector, g is a bounded, continuous function, and p is the critical Sobolev exponent $\frac{2N}{N-2}$, where $N \geq 3$ is the dimension. The first-order term $\vec{c} \cdot \nabla u$, which typically arises as a flow term, precludes us from using the calculus of variations. In this situation, the main techniques for proving existence of solutions have been topological. However, due to the nature of the nonlinearity, the corresponding operator is not compact and Leray-Schauder degree theory does not apply. In this talk, I will discuss the methods of concentration compactness, as developed by P.L.Lions, the (S_+) degree theory of Skrypnik, and how these new techniques can be used to prove existence for Equation 0.1. These methods are also applicable to a range of elliptic systems. The development of new methods to study these problems is important both theoretically and for applications, as most equations and systems are not variational. Future work, including related quasilinear problems, will be mentioned. This talk is joint work with M. Chhetri, P. Drabek, and S. Robinson.

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