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Diffusion and Mixing by Fluid Flow

ABSTRACT:

We consider the advection-diffusion equation

$$\phi_t + Au \cdot \nabla \phi = \Delta \phi, \quad \phi(0, x) = \phi_0(x)$$

in \mathbb{R}^2 , where u is a periodic incompressible flow with a large amplitude A . We characterize all u that are optimally mixing in the sense that for any compactly supported initial datum ϕ_0 and any $t > 0$,

$$\|\phi(t, \cdot)\|_\infty \rightarrow 0 \quad \text{as } A \rightarrow \infty.$$

Our characterization is expressed in terms of simple geometric and spectral conditions on the flow. Moreover, if this convergence holds, it is uniform for ϕ_0 in the unit ball in $L^p(\mathbb{R}^2)$, $1 \leq p < \infty$, and $\|\cdot\|_\infty$ can be replaced by any $\|\cdot\|_q$, $q > p$. Extensions to higher dimensions and applications to reaction-advection-diffusion equations will also be discussed.