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“Motion of concentrations in Ginzburg-Landau theory”

Abstract:

Complex Ginzburg-Landau equations comprise a large family of phase transition equations that describe quantum mechanical effects ”on the large”. Among other physical phenomena, they serve as excellent models for both superconductors and superfluids. The sets where the wave function vanish are commonly called vortices and contain the most relevant physical information of the system. For example an applied magnetic field pierces the material bulk at the location of each vortex. The strength of a vortex can be controlled by a single non-dimensional Ginzburg-Landau parameter, and there has been a lot of recent interest, dating from the breakthrough work of Bethuel-Brezis-Helein, in understanding the behavior of solutions as this parameter becomes asymptotically large. Such problems are closely related to the study of harmonic map relaxation problems.

Once dynamics are turned on, the motion of these vortices reduce to surprisingly simple ODE’s; however, the rigorous proof of these motion laws entails studying an energy that generically blows up. I will discuss the behavior of such concentrations in the asymptotic limit in both dissipative and dispersive systems. I will also describe current work towards describing the behavior of vortices for finite Ginzburg-Landau parameter.