MATH 8402: Assignment 2

Problem 1. Let us consider a material curve, $\mathcal{C}_0 : \mathbf{X} = \mathbf{X}(\tau)$, in the reference configuration of a body $\Omega_0 \subset \mathbf{R}^3$; $I \subset \mathbf{R}$ is an open interval, and $\tau \in I$, is the parameter of the curve. Suppose that Ω_0 is mapped into $\Omega \subset \mathbf{R}^3$ by the deformation map $\mathbf{x} = \Phi(\mathbf{X})$. Let $\mathcal{C}$ denote the deformed curve.

- Find the relationship between the unit tangent vectors $\hat{\mathbf{T}}$ and $\hat{\mathbf{t}}$ to the reference and deformed curves, at points $\mathbf{X}$ and $\Phi(\mathbf{X})$, respectively,
- Let dL and dl denote arc lengths of the reference and deformed curves, respectively. Find their relationship.

Problem 2. Obtain the conditions on the deformation gradient $F = \nabla\Phi(\mathbf{X})$ so that:

- No extension occurs on a specified direction;
- The angle between a pair of specified directions is unchanged;
- No change of area takes place in the plane orthogonal to a specified direction;
- No change of volume occurs.

Problem 3. In connection with problem 1, and assuming a time dependent deformation map $\mathbf{x} = \Phi(\mathbf{X}, t)$,

- Find the material derivative of the arc lenght dl;
- Let $\mathcal{C}_t$ and $\mathcal{D}_t$ denote two intersecting curves in the deformed configuration. Find the material derivative of the angle, θ between the two curves.

Problem 4. In its reference configuration a body contains a spherical cavity of center O and radius A, filled with explosive. The explosive is detonated at $t = 0$ and produces a spherically symmetric motion of the body given by

$$\mathbf{x} = \frac{r}{R}\mathbf{X}, \ r = f(R, t),$$

where $R = |\mathbf{X}|$, $r = |\mathbf{x}|$, and the referential and current positions, $\mathbf{X}$ and $\mathbf{x}$, are both taken relative to O as origin. If the motion is isochoric and the cavity radius at time t is a, show that

$$f(R, t) = (R^3 + a^3 - A^3)^{\frac{1}{3}}.$$

Determine the velocity and acceleration in the spatial description.

The assignment is due on Wednesday, February 27 .