

MATH 8402: Exercises

Problem 1. Consider the deformation with gradient

$$F = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Obtain the corresponding matrices of the polar decomposition $F = RU$.

Problem 2. A body contains two families of fibers whose orientations \mathbf{L} and \mathbf{M} are preserved during deformation, that is, there are positive scalars λ and μ such that

$$F\mathbf{L} = \lambda\mathbf{L}, \quad F\mathbf{M} = \mu\mathbf{M}.$$

In the reference configuration a line element is parallel to $\mathbf{L} + \alpha\mathbf{M}$. Find the change of length ratio of the line element under the deformation.

Problem 3. Define:

- *Transformation in observer;*
- *Objective scalar, objective vector and objective tensor* with respect to the transformation.

Problem 4. Given the scalar field $\phi(\mathbf{X}, t)$ and the deformation map $\mathbf{x} = \Phi(\mathbf{X}, t)$, $\mathbf{X} \in \Omega_0$, recall that we refer to *Lagrangian* or *material time derivative* of ϕ , the time derivative obtained by holding \mathbf{X} fixed. (Likewise, the *Eulerian* or *spatial* is obtained by holding \mathbf{x} fixed).

- Suppose that ϕ is an objective scalar under a transformation in observer. Show that the material time derivative of ϕ is also an objective scalar under the transformation;
- Is the Eulerian time derivative an objective scalar?
- Is $L = \nabla_{\mathbf{x}}\Phi$ an objective tensor?

Problem 5. Consider the deformation map defined in Cartesian coordinates by

$$x_1 = X_1 e^{t^2}, \quad x_2 = X_2 e^t, \quad x_3 = X_3.$$

Determine the *trajectories* and *streamlines* of the motion.

Problem 6. Let $T(\mathbf{x}, t)$ denote the (symmetric) Cauchy stress tensor. The eigenvalues and eigenvectors of T are known as *principal stress* and *principal directions*, respectively. Find the principal stresses and principal directions of

$$T = \tau(\mathbf{e}_1 \otimes \mathbf{e}_3 + \mathbf{e}_3 \otimes \mathbf{e}_1),$$

where τ is constant.

The notation used in the problems is the same as in class.

This list illustrates the type of problems to expect in the midterm examination.