# The Earth's Glacial Cycles

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School of Mathematics
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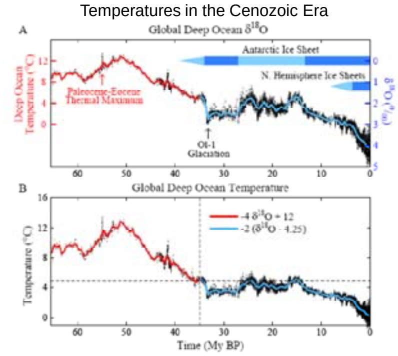
http://www.tqnyc.org/NYC052141/beginningpage.html

## Glacial Cycles

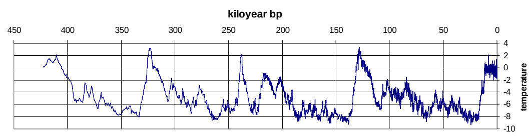
Temperatures in the Cenozoic Era




Hansen, et al, 2008, p. 7

## Glacial Cycles

Recent Temperature Cycles




Note the period of about 100 kyr.

## Glacial Cycles

What Causes Glacial Cycles?

**Widely Accepted Hypothesis**

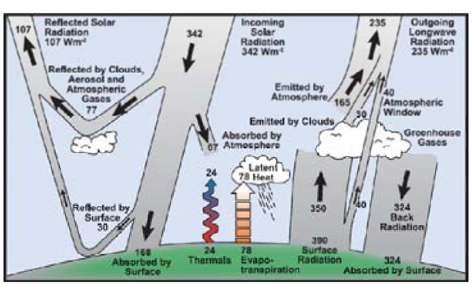
The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work is not fully understood.

## Glacial Cycles

Heat Balance

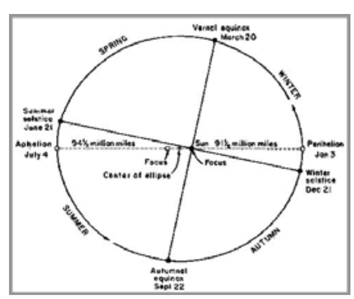



*Historical Overview of Climate Change Science*, IPCC AR4, p.96
http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf

## Glacial Cycles

Milankovitch Cycles

Eccentricity

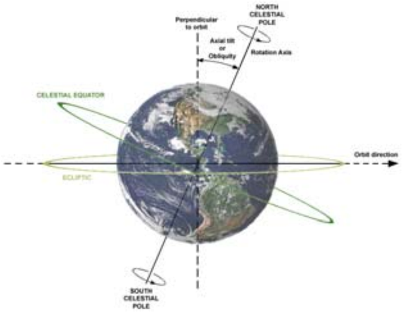



http://www.crrel.usace.army.mil/permafrosttunnel/Ice_Age_Earth_Orbit.jpg

## Glacial Cycles

### Milankovitch Cycles

**Obliquity**

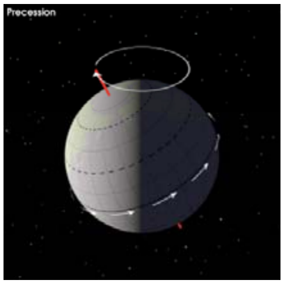



http://upload.wikimedia.org/wikipedia/commons/6/61/AxialTiltObliquity.png

## Glacial Cycles

### Milankovitch Cycles

**Precession**




http://earthobservatory.nasa.gov/Library/Giants/Milankovitch/milankovitch_2.html

## Glacial Cycles

### Milankovitch Cycles

Perihelion: 91.5
Aphelion: 94.5

Change in radius:
3/93 = 3.2%

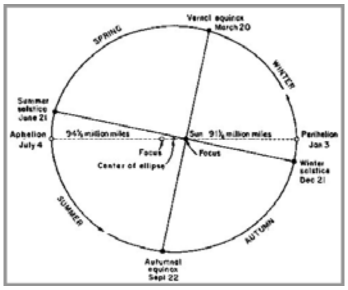
Change in insolation: 6.4%

Six percent less insolation in the southern winter than the northern winter.

6.4% of 342 $Wm^2$ =

**22 $Wm^{-2}$**

**Eccentricity**




## Glacial Cycles

### Global Annual Average Insolation

Solar intensity at distance $r$ from the sun:

$$Q(t) = \frac{k_1}{r(t)^2}$$

Angular momentum:

$$\Omega = r^2 \dot{\theta}$$

$$Q(t) = \frac{k_1 \dot{\theta}}{\Omega}$$

Mean annual solar input ( $T$ = one year):

$$\bar{Q} = \frac{1}{T}\int_0^T Q(t)\,dt = \frac{k_1}{T\Omega}\int_0^T \dot{\theta}\,dt = \frac{2\pi k_1}{T\Omega}$$

## Glacial Cycles

### Global Annual Average Insolation

Kepler's Third Law: $T = k_2 a^{-3/2}$ $a$ = semimajor axis

Mean annual solar input:

$$\bar{Q} = \frac{k_3 a^{3/2}}{\Omega}$$

Derived from Kepler: $1 - e^2 = k_4 a \Omega^2$ $e$ = eccentricity

Finally:

$$\boxed{\bar{Q} = \frac{k_5 a^2}{\sqrt{1-e^2}}}$$

## Glacial Cycles

### Global Annual Average Insolation

Laskar: $\bar{Q} = \frac{k_5 a^2}{\sqrt{1-e^2}}$

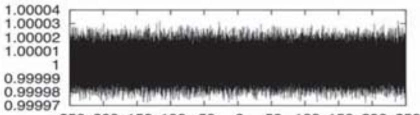


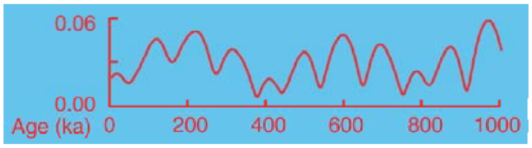

**Fig. 11.** Variation of the semi-major axis of the Earth–Moon barycenter (in AU) from −250 to +250 Myr.

$$\delta a \approx 0.00005 \;\leftrightarrow\; \delta\bar{Q} \approx 0.0001 \;\leftrightarrow\; 0.03\,\mathrm{Wm^{-2}}$$

What about eccentricity?

## Glacial Cycles
**Milankovitch Cycles**

**Eccentricity**



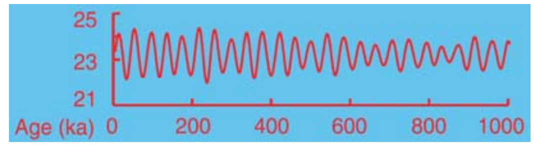

Note periods of about 100 kyr and 400 kyr.

$$0 < e < 0.05 \quad \leftrightarrow \quad 1 < \left(1-e^2\right)^{-1/2} < 1.00125 \quad \leftrightarrow \quad \delta\bar{Q} \approx 0.0025 \quad \leftrightarrow \quad 0.9\,\mathrm{Wm}^{-2}$$

**Zachos, et al, "Trends, Rhythms, and Aberrations in Global Climate 65 Ma to Present". *Science* 292** (2001)**, 687.**

## Glacial Cycles
**Milankovitch Cycles**

**Obliquity**

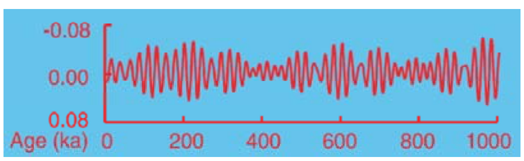



Note period of about 41 kyr.

**Zachos, et al, "Trends, Rhythms, and Aberrations in Global Climate 65 Ma to Present". *Science* 292** (2001)**, 687.**

## Glacial Cycles
**Milankovitch Cycles**

**Precession**

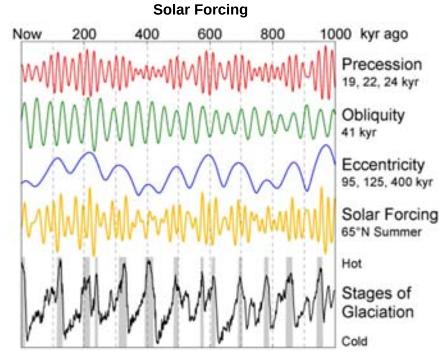



Note period of about 23 kyr.

**Zachos, et al, "Trends, Rhythms, and Aberrations in Global Climate 65 Ma to Present". *Science* 292** (2001)**, 687.**

## Glacial Cycles
**Milankovitch Cycles**

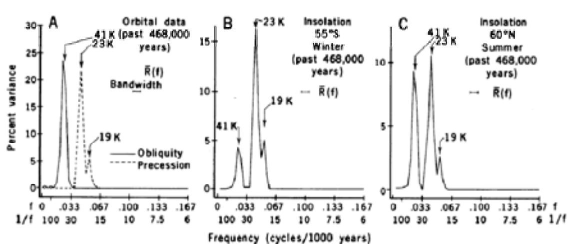
**Solar Forcing**




http://en.wikipedia.org/wiki/Milankovitch_cycles

## Glacial Cycles
**Milankovitch Cycles**

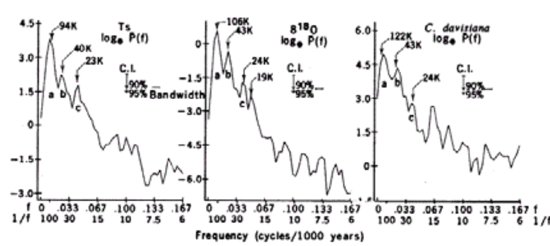
**Solar Forcing (Hays, et al)**




Hays, *et al*, *Science* **194** (1976), p. 1125

## Glacial Cycles
**Milankovitch Cycles**

**Climate Response, Hays, et al**




Three different temperature proxies from sea sediment data.

Hays, *et al*, *Science* **194** (1976), p. 1127

## Glacial Cycles

**Hays, et al, Summary**

**1)** Three indices of global climate have been monitored in the record of the past 450,000 years in Southern Hemisphere ocean-floor sediments.

**2)** ... climatic variance of these records is concentrated in three discrete spectral peaks at periods of 23,000, 42,000, and approximately 100,000 years. These peaks correspond to the dominant periods of the earth's solar orbit, and contain respectively about 10, 25, and 50 percent of the climatic variance.

Hays, *et al*, *Science* **194** (1976), p. 1131

## Glacial Cycles

**Hays, et al, Summary**

**3)** The 42,000-year climatic component has the same period as variations in the obliquity of the earth's axis and retains a constant phase relationship with it.

**4)** The 23,000-year portion of the variance displays the same periods (about 23,000 and 19,000 years) as the quasiperiodic precession index.

**5)** The dominant, 100,000-year climatic component has an average period close to, and is in phase with, orbital eccentricity. Unlike the correlations between climate and the higher-frequency orbital variations (which can be explained on the assumption that the climate system responds linearly to orbital forcing), an explanation of the correlation between climate and eccentricity probably requires an assumption of nonlinearity.

Hays, *et al*, *Science* **194** (1976), p. 1131

## Glacial Cycles

**Hays, et al, Summary**

**6)** It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages.

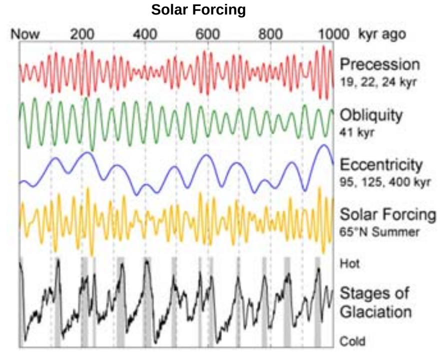
**7)** A model of future climate based on the observed orbital-climate relationships, but ignoring anthropogenic effects, predicts that the long-term trend over the next seven thousand years is toward extensive Northern Hemisphere glaciation.

Hays, *et al*, *Science* **194** (1976), p. 1131

## Glacial Cycles

**Milankovitch Cycles**

**Solar Forcing**



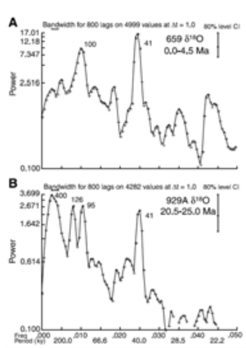

http://en.wikipedia.org/wiki/Milankovitch_cycles

## Glacial Cycles

**Climate Response (Zachos, et al)**

**A.** Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41Kyr and 100 Kyr.

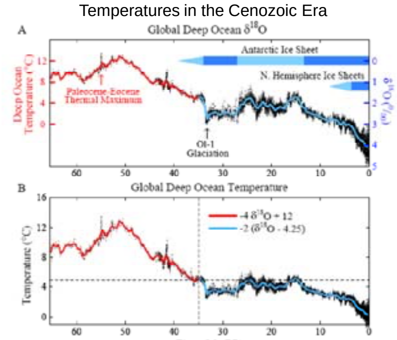
**B.** Power spectrum of climate for the period 25 Myr bp to 20.5 Myr bp. Note the new peak at 400 Kyr and the "split" peaks at 126Kyr and 95 Kyr.




Zachos, *et al*, *Science* **292** (2001), p. 689

## Glacial Cycles

Temperatures in the Cenozoic Era




Hansen, et al, 2008, p. 7

## Glacial Cycles

**Are Milankovitch Cycles Sufficient?**

**Rind, et al, used a global circulation model to try to predict the last ice age using the Milankovitch insolation data. It failed:**

"The results show that the model fails to maintain snow cover through the summer at locations of suspected initiation of the major ice sheets, despite the reduced summer and fall insolation."

"The experiments indicate there is a wide discrepancy between the model's response to Milankovitch perturbations and the geophysical evidence of ice sheet initiation."

Rind, *et al*, *J. Geophysical Research* **94** (1989), p. 12851

## Glacial Cycles

**Are Milankovitch Cycles Sufficient?**

**Rind, continued.**

"If the model results are correct, it indicates that the growth of ice occurred in an extremely ablative environment, and thus demanded some complicated strategy, or else some other climate forcing occurred in addition to the orbital variation influence (and $CO_2$ reduction), which would imply we do not really understand the cause of the ice ages and the Milankovitch connection. If the model is not nearly sensitive enough to climate forcing, it could have implications for projections of future climate change."

Rind, *et al*, *J. Geophysical Research* **94** (1989), p. 12851

## Glacial Cycles

**Are Milankovitch Cycles Sufficient?**

A Milankovitch Skeptic

Climate variability in this range of periods is difficult to distinguish from a form of random walk with small superimposed deterministic elements. Evidence that Milankovitch forcing "controls" the records, in particular the 100 ka glacial/interglacial, is very thin and somewhat implausible, …

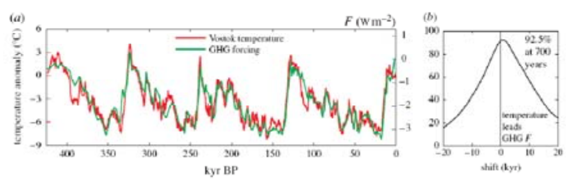
Carl Wunsch

Carl Wunsch, 2004, Quantitative estimate of the Milankovitch-forced contribution to Observed Quaternary climate change, *Quaternary Science Reviews* **23** (2004), 1001–1012

## Glacial Cycles

**What about $CO_2$?**

$CO_2$ lags temperature in the Vostok data.




Hansen, et al, 2007, *Phil. Trans. R. Soc. A* **365**, 1925–1954

## Glacial Cycles

**Does $CO_2$ Provide the Feedback?**

$$c\frac{dT}{dt} = S(t) + G(C) - \sigma T^4,$$
$$\frac{dC}{dt} = V - (W_0 + W_1 C) + \beta (C_{max} - C)\max\left(\frac{dT}{dt} - \varepsilon, 0\right).$$

Andrew Hogg

Hogg 2008, *Geophysical Research Letters* **35**.

## Glacial Cycles

**Hogg's Model**

$$c\frac{dT}{dt} = S(t) + G(C) - \sigma T^4,$$
$$\frac{dC}{dt} = V - (W_0 + W_1 C) + \beta (C_{max} - C)\max\left(\frac{dT}{dt} - \varepsilon, 0\right).$$

volcanoes: $V$; weathering: $(W_0 + W_1 C)$; CO2 outgassing: $\beta (C_{max} - C)\max\left(\frac{dT}{dt} - \varepsilon, 0\right)$

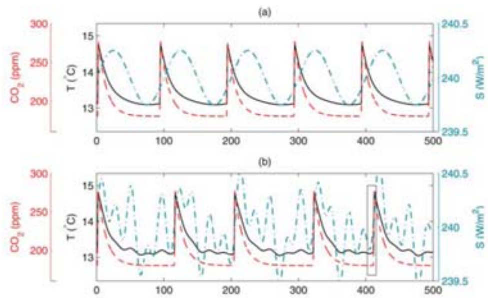
$$S(t) = \bar{S} + \sum_i S_i \sin\left(\frac{2\pi t}{\Gamma_i}\right) \quad \text{insolation}$$

$$G(C) = \bar{G} + A\ln\left(\frac{C}{C_0}\right) \quad \text{greenhouse forcing}$$

Should weathering depend on temperature?

## Glacial Cycles

**Hogg's Model**




Hogg 2008, *Geophysical Research Letters* **35**.

## Glacial Cycles

**Budyko's Ice Line Model**

$$K\frac{dT}{dt} = \boxed{Qs(y)}\left(1-\alpha(T)(y)\right) - I(T)(y) + H(T)(y)$$

The annual global average insolation is $Q$.
The annual average insolation as a function of latitude $\theta$,
where $y = \sin\theta$, is $Qs(y)$.
$Q$ is largely determined by the eccentricity, but $s(y)$ is determined from a combination of the other orbital elements.

What about $s(y,t)$?

## Glacial Cycles

*Are there PhD thesis projects?*

Three suggestions:

1. Hogg's model with weathering as a function of $CO_2$ *and* temperature.
2. Budyko's model with insolation as a function of latitude *and* geologic time.
3. Budyko's model with a Hogg-like $CO_2$ feedback.