Milankovitch Cycles

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Seminar on the Mathematics of Climate Change
School of Mathematics
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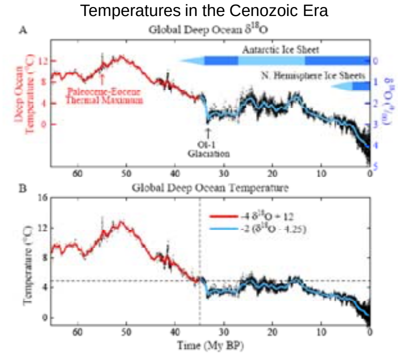
http://www.tqnyc.org/NYC052141/beginningpage.html

Milankovitch Cycles

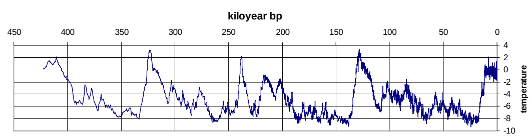
Temperatures in the Cenozoic Era



Hansen, et al, 2008, p. 7

Milankovitch Cycles

Recent Temperature Cycles



Note the period of about 100 kyr.

Milankovitch Cycles

What Causes Glacial Cycles?

Widely Accepted Hypothesis

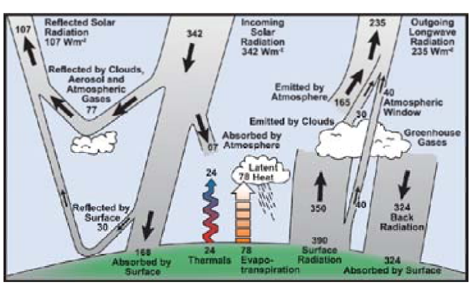
The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work is not fully understood.

Milankovitch Cycles

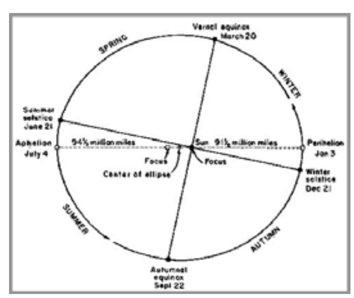
Heat Balance



Historical Overview of Climate Change Science, IPCC AR4, p.96
http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf

Milankovitch Cycles

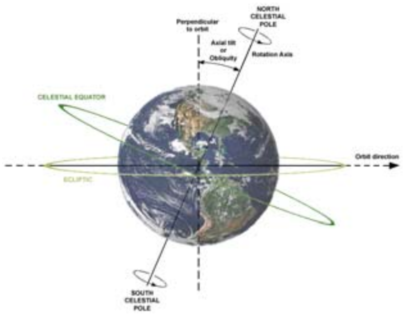
Eccentricity



http://www.crrel.usace.army.mil/permafrosttunnel/Ice_Age_Earth_Orbit.jpg

Milankovitch Cycles

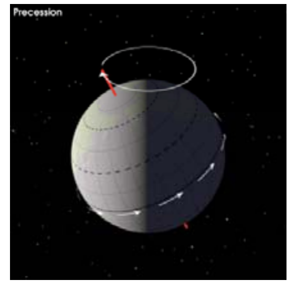
Obliquity



http://upload.wikimedia.org/wikipedia/commons/6/61/AxialTiltObliquity.png

Milankovitch Cycles

Precession



http://earthobservatory.nasa.gov/Library/Giants/Milankovitch/milankovitch_2.html

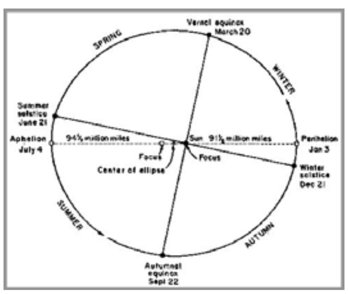
Milankovitch Cycles

Eccentricity

Perihelion: $91.5\text{x}10^6$ mi
Aphelion: $94.5\text{x}10^6$ mi

Semimajor axis: $93\text{x}10^6$ mi

Eccentricity: 1.5/93 = 0.016



Milankovitch Cycles

Global Annual Average Insolation

Solar output: K Watts

Solar intensity at distance r from the sun:

$$Q(t) = \frac{K}{4\pi r(t)^2} \quad \text{Wm}^{-2}$$

Cross section of Earth: πr_E^2 m^2

Global solar input: $\dfrac{K r_E^2}{4 r(t)^2}$ W

Total annual solar input (P = one year (in seconds)):

$$\int_0^P \frac{K r_E^2}{4 r(t)^2} dt = \frac{K r_E^2}{4} \int_0^P \frac{dt}{r(t)^2} \quad \text{Joules}$$

Milankovitch Cycles

Global Annual Average Insolation

Specific angular momentum (angular momentum per unit mass):

$$\Omega = r^2 \dot{\theta} \quad \text{m}^2\text{s}^{-1}$$

Total annual solar input:

$$\frac{K r_E^2}{4} \int_0^P \frac{dt}{r(t)^2} = \frac{K r_E^2}{4} \int_0^P \frac{\dot{\theta} dt}{\Omega} = \frac{K r_E^2}{4\Omega} \int_0^{2\pi} d\theta = \frac{\pi K r_E^2}{2\Omega} \quad \text{Joules}$$

Mean annual solar input:

$$\frac{\pi K r_E^2}{2P\Omega} \quad \text{Watts}$$

Mean annual solar intensity on the Earth's surface:

$$\frac{\pi K r_E^2}{2P\Omega} \cdot \frac{1}{4\pi r_E^2} = \frac{K}{8P\Omega} \quad \text{Wm}^{-2}$$

Milankovitch Cycles

Global Annual Average Insolation

Kepler's Third Law:

$P \sim a^{-3/2}$ a = semimajor axis

Derived from Kepler:

$1 - e^2 \sim a\Omega^2$ e = eccentricity

Mean annual solar intensity:

$$\frac{K}{8P\Omega} = \frac{\hat{K} a^{3/2} a^{1/2}}{\sqrt{1-e^2}} = \frac{\hat{K} a^2}{\sqrt{1-e^2}} \quad \text{Wm}^{-2}$$

Milankovitch Cycles

Global Annual Average Insolation

Laskar: $\frac{\hat{K}a^2}{\sqrt{1-e^2}}$

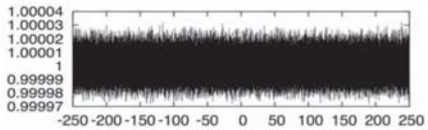
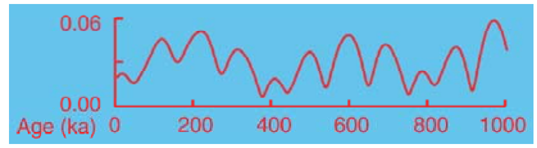


Fig. 11. Variation of the semi-major axis of the Earth–Moon barycenter (in AU) from −250 to +250 Myr.

Semi major axis does not change much.

Milankovitch Cycles

Eccentricity



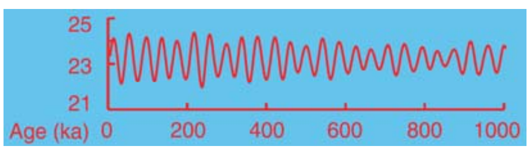
The effect due to eccentricity is more significant.

Note periods of about 100 Kyr and 400 Kyr.

Zachos, et al, "Trends, Rhythms, and Aberrations in Global Climate 65 Ma to Present". ***Science*** **292** (2001)**, 687.**

Milankovitch Cycles

Obliquity

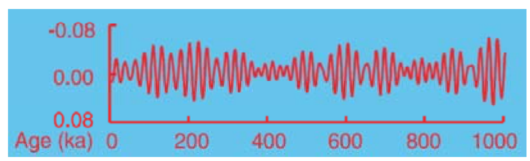


Note period of about 41 Kyr.

Zachos, et al, "Trends, Rhythms, and Aberrations in Global Climate 65 Ma to Present". ***Science*** **292** (2001)**, 687.**

Milankovitch Cycles

Precession

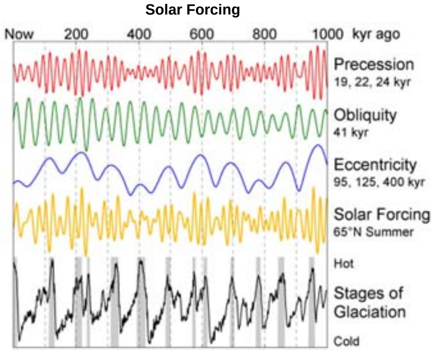


Note period of about 23 Kyr.

Zachos, et al, "Trends, Rhythms, and Aberrations in Global Climate 65 Ma to Present". ***Science*** **292** (2001)**, 687.**

Milankovitch Cycles

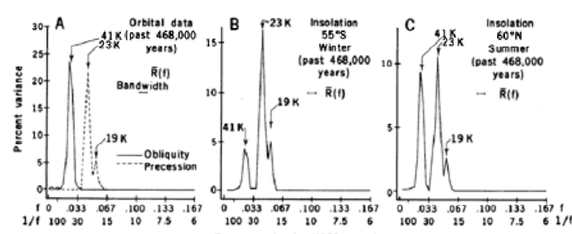
Solar Forcing



http://en.wikipedia.org/wiki/Milankovitch_cycles

Milankovitch Cycles

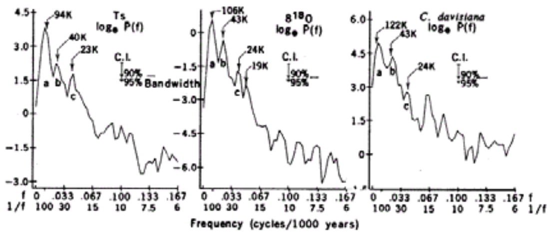
Solar Forcing (Hays, et al)



Hays, *et al*, *Science* **194** (1976), p. 1125

Milankovitch Cycles

Climate Response, Hays, et al



Three different temperature proxies from sea sediment data.

Hays, *et al*, *Science* **194** (1976), p. 1127

Milankovitch Cycles

Hays, et al, Summary

1) Three indices of global climate have been monitored in the record of the past 450,000 years in Southern Hemisphere ocean-floor sediments.

2) ... climatic variance of these records is concentrated in three discrete spectral peaks at periods of 23,000, 42,000, and approximately 100,000 years. These peaks correspond to the dominant periods of the earth's solar orbit, and contain respectively about 10, 25, and 50 percent of the climatic variance.

Hays, *et al*, *Science* **194** (1976), p. 1131

Milankovitch Cycles

Hays, et al, Summary

3) The 42,000-year climatic component has the same period as variations in the obliquity of the earth's axis and retains a constant phase relationship with it.

4) The 23,000-year portion of the variance displays the same periods (about 23,000 and 19,000 years) as the quasiperiodic precession index.

5) The dominant, 100,000-year climatic component has an average period close to, and is in phase with, orbital eccentricity. Unlike the correlations between climate and the higher-frequency orbital variations (which can be explained on the assumption that the climate system responds linearly to orbital forcing), an explanation of the correlation between climate and eccentricity probably requires an assumption of nonlinearity.

Hays, *et al*, *Science* **194** (1976), p. 1131

Milankovitch Cycles

Hays, et al, Summary

6) It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages.

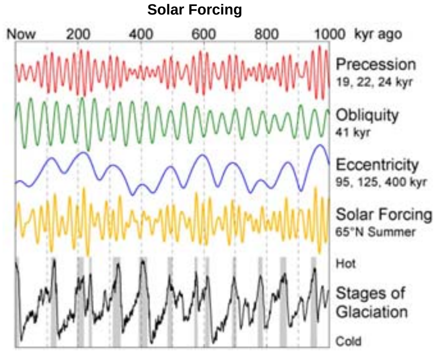
7) A model of future climate based on the observed orbital-climate relationships, **but ignoring anthropogenic effects**, predicts that the long-term trend over the next seven thousand years is toward **extensive Northern Hemisphere glaciation***.

*Quoted by George Will, Washington Post, February 5, 2009

Hays, *et al*, *Science* **194** (1976), p. 1131

Milankovitch Cycles

Solar Forcing



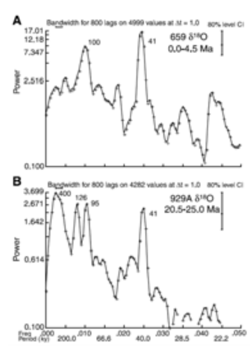
http://en.wikipedia.org/wiki/Milankovitch_cycles

Milankovitch Cycles

Climate Response (Zachos, et al)

A. Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41Kyr and 100 Kyr.

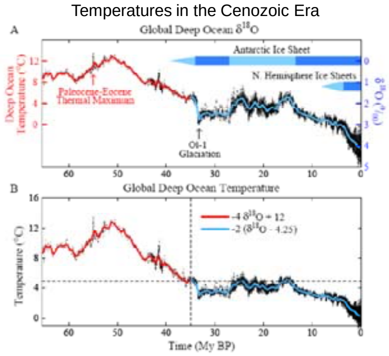
B. Power spectrum of climate for the period 25 Myr bp to 20.5 Myr bp. Note the new peak at 400 Kyr and the "split" peaks at 126Kyr and 95 Kyr.



Zachos, *et al*, *Science* **292** (2001), p. 689

Milankovitch Cycles

Temperatures in the Cenozoic Era

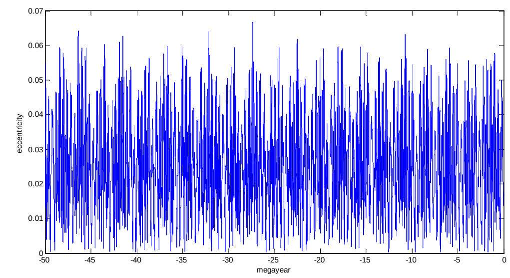


Hansen, et al, 2008, p. 7

Milankovitch Cycles

The cycles have not changed much.

Eccentricity

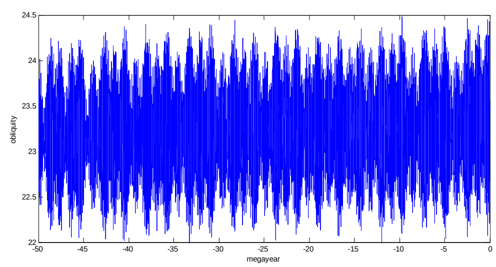


Laskar

Milankovitch Cycles

The cycles have not changed much.

Obliquity



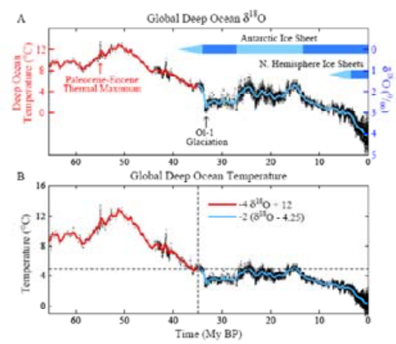
Laskar

Milankovitch Cycles

The Milankovitch cycles have not changed much, but the climate response has changed a lot.

Not just a nonlinear response.

But that's a different story.



Hansen, et al, 2008, p. 7

Milankovitch Cycles

Summary

The solar forcing, defined as the maximum insolation at latitude 65° N, is dominated by precession, followed by obliquity, followed by eccentricity.

The climate response is dominated by
eccentricity, followed by obliquity, followed by precession (Hays)
OR
obliquity, followed by eccentricity, with negligible precession (Zachos).

The explanation is that there are nonlinear feedbacks.

The total annual solar input depends mainly on eccentricity, and a little bit on semimajor axis, but not at all on obliquity or precession.

Is there another explanation?

Milankovitch Cycles

Budyko's Ice Line Model

$$R\frac{dT}{dt} = \boxed{Qs(y)}\left(1-\alpha(T)(y)\right) - I(T)(y) + H(T)(y)$$

The annual global average insolation is Q .
The annual average insolation as a function of latitude θ,
where $y = \sin\theta$, is $Qs(y)$.
Q is largely determined by the eccentricity, but $s(y)$ is determined from a combination of the other orbital elements.

What is $s(y)$ as a function of obliquity and precession?

Stay tuned.

Milankovitch Cycles

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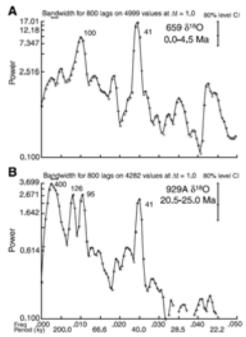

Zachos, *et al*, *Science* **292** (2001), p. 689