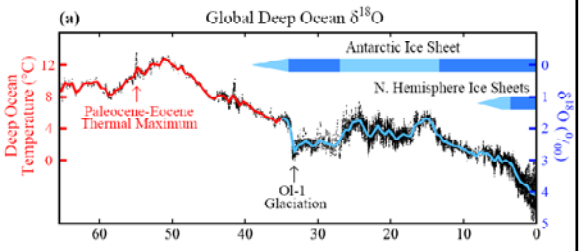
Recent Developments in the Theory of Glacial Cycles

Richard McGehee

Seminar on the Mathematics of Climate Change
School of Mathematics
October 6, 2010

Glacial Cycles

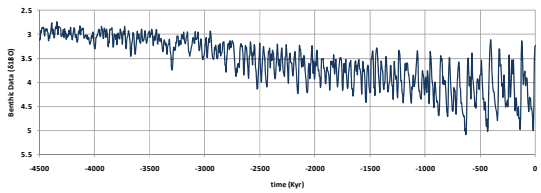
Temperatures in the Cenozoic Era



Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

Glacial Cycles

^{18}O in Foraminifera Fossils During the Past 4.5 Myr



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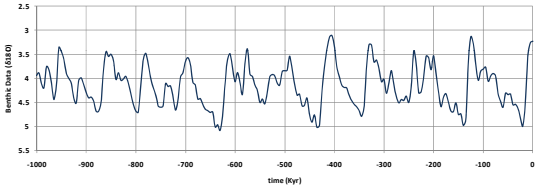
Glacial Cycles

^{18}O in Foraminifera Fossils During the Past 1.0 Myr



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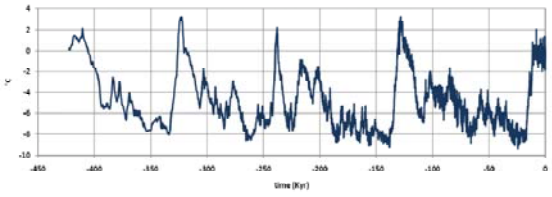
Glacial Cycles

Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data



J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* **399**, 429-436.

Glacial Cycles

What Causes Glacial Cycles?

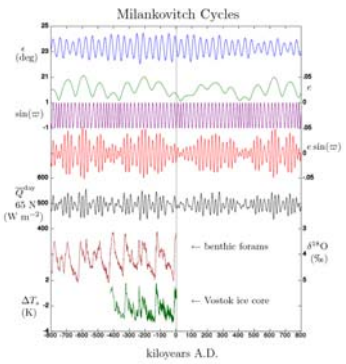
Widely Accepted Hypothesis

The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work is not fully understood.

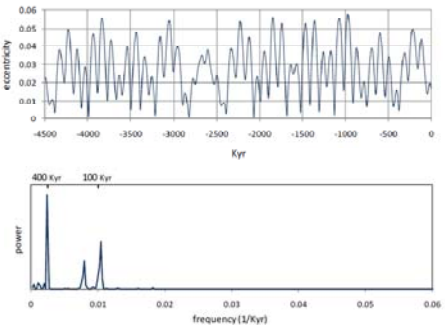
Glacial Cycles



http://en.wikipedia.org/wiki/Milankovitch_cycles

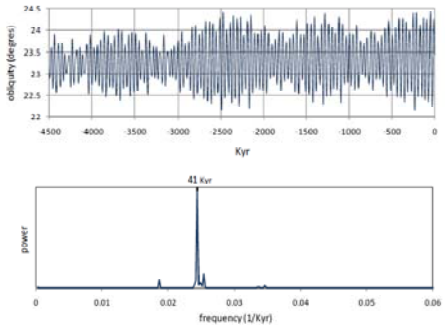
Glacial Cycles

Eccentricity



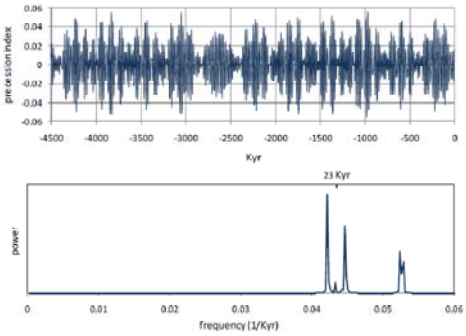
Glacial Cycles

Obliquity



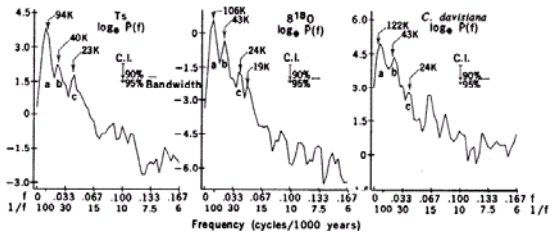
Glacial Cycles

Precession Index



Glacial Cycles

Climate Response, Hays, et al

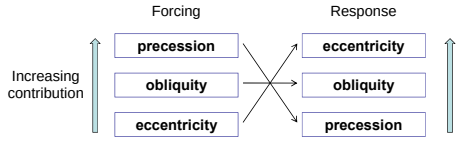


Three different temperature proxies from sea sediment data.

Hays, *et al*, *Science* **194** (1976), p. 1127

Glacial Cycles

Hays Summary



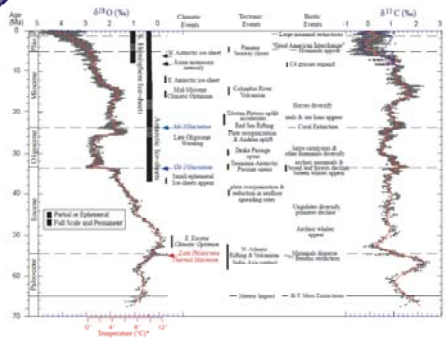
The explanation is that there are nonlinear feedbacks.

Is there another explanation?

Hays, *et al*, *Science* **194** (1976), p. 1127

Glacial Cycles

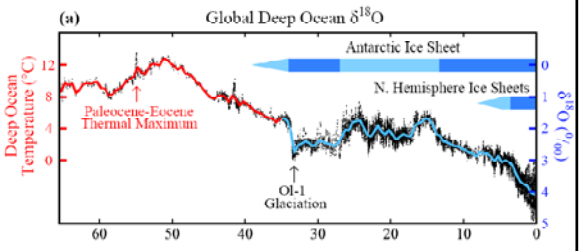
Cenozoic Era



Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Temperatures in the Cenozoic Era



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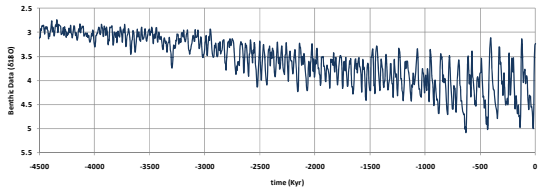
Glacial Cycles

^{18}O in Foraminifera Fossils During the Past 4.5 Myr



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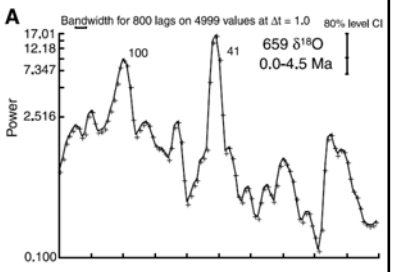
Glacial Cycles

Climate Response (Zachos, et al)

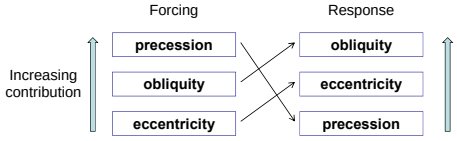
Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41Kyr and 100 Kyr.



Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Zachos Summary



Nonlinear effects?

Forcing is defined as the maximum insolation at latitude 65° N.

Is there another definition of forcing?

Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Energy Balance

Global annual average insolation

$$Q = Q(e) = \frac{Q_0}{\sqrt{1-e^2}}$$

Distribution of annual average insolation with latitude

$$s(y) = s(y,\beta) = \frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\cos\gamma - y\cos\beta\right)^2}\,d\gamma$$

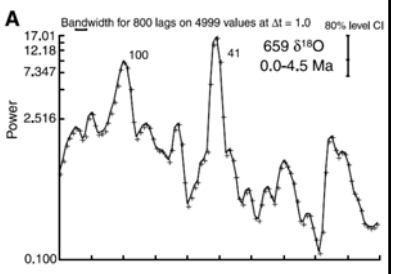
e = eccentricity
β = obliquity

Note that Q depends only on eccentricity, $s(y)$ depends only on obliquity, and nothing depends on precession.

Glacial Cycles

Climate Response (Zachos, et al)

Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41Kyr and 100 Kyr.



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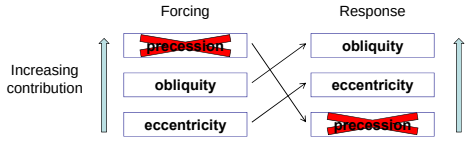
Glacial Cycles

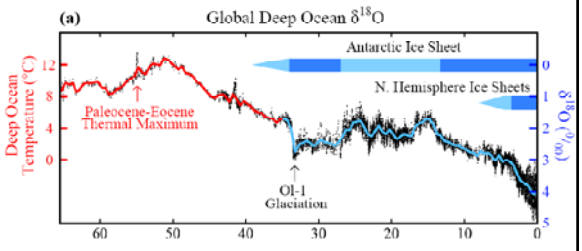
Energy Balance

If we assume that glaciation depends on annual average insolation instead of insolation at summer solstice, then forcing and response are aligned.



Glacial Cycles

Temperatures in the Cenozoic Era

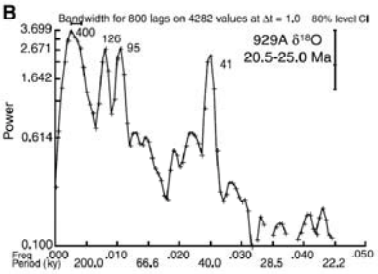


Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

Glacial Cycles

Climate Response (Zachos, et al)

Power spectrum of climate for the period 20.5-25.0 Myr. Note the peak at 400.



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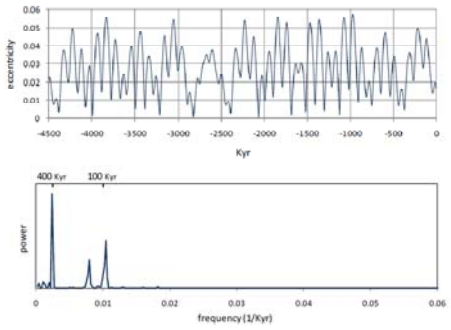
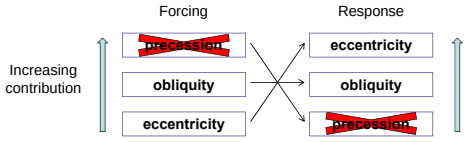
Glacial Cycles

Eccentricity



Glacial Cycles

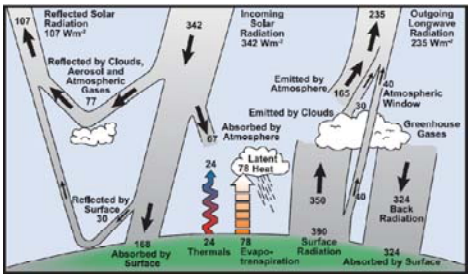
Zachos 20 Myr ago



Maybe 65° N at summer solstice isn't the right forcing.

Glacial Cycles

Heat Balance



Historical Overview of Climate Change Science, IPCC AR4, p.96
`http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf`

Glacial Cycles

Ice Albedo Feedback Model

$$R\frac{\partial T}{\partial t} = \underbrace{Qs(y)}_{\text{insolation}}\underbrace{(1-\alpha(y,\eta))}_{\text{albedo}} - \underbrace{(A+BT)}_{\text{outward radiation}} + \underbrace{C(\overline{T}-T)}_{\text{heat transport}}$$

y = sine of latitude
$T(y)$ = annual mean temperature
$Qs(y)$ = annual mean insolation
Q = global annual mean insolation

This equation has a stable equilibrium consisting of polar ice caps.

The latitude of the equilibrium ice boundary and the equilibrium global annual mean temperature are functions of the parameters.

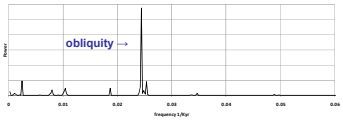
K.K. Tung, *Topics in Mathematical Modeling*, Princeton (2007), Chapt 8

Glacial Cycles

Power Spectra

Note dominance of the obliquity signal.

ice-albedo model



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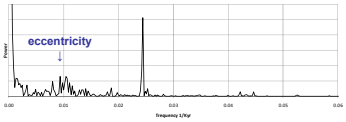
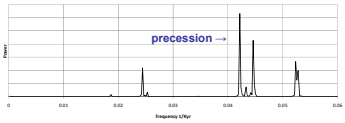


Insolation at 65° N at summer solstice

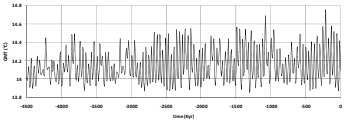


Glacial Cycles

Annual Global Mean Temperature

The data are not duplicated by either method, but note the spectral similarities.

ice-albedo model



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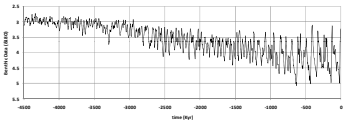

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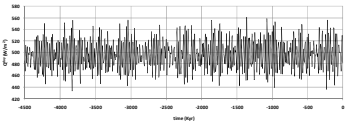

Glacial Cycles

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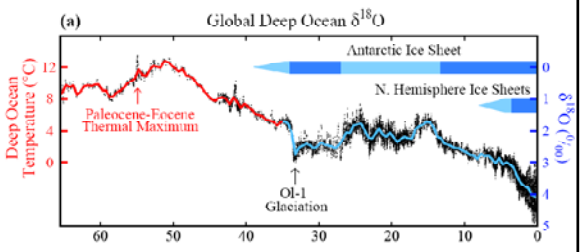
If the albedo is constant (i.e., independent of latitude, then the global mean temperature depends only on eccentricity, not obliquity.

Prediction: Analysis of the climate 40-50 Myr ago should show increased eccentricity signal (400 and 100 Kyr) and reduced obliquity signal (41 Kyr).

(current research)

Glacial Cycles

Temperatures in the Cenozoic Era



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Glacial Cycles

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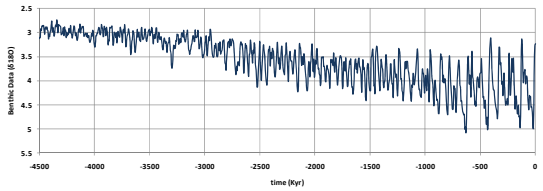

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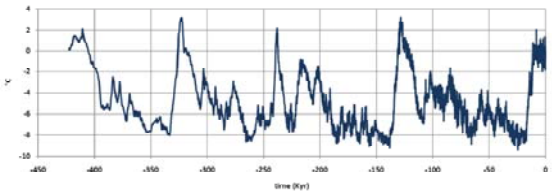
Glacial Cycles

Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data



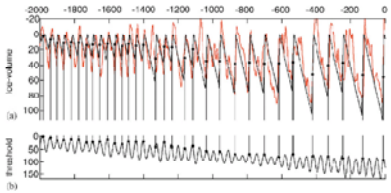
J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* **399**, 429-436.

Glacial Cycles

Triggering Mechanism

Idea (Huybers, Paillard): The glaciers tend to grow. From time-to-time, the obliquity reaches a critical value triggering a melt-down.

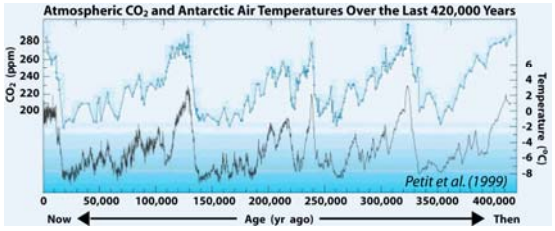
$$V_t = V_{t-1} + \eta_t \quad \text{and if } V_t \geq T_t \text{ terminate,}$$
$$T_t = at + b - c\theta'_t.$$



Peter Huybers, *Quaternary Science Reviews* **26** (2007)

Glacial Cycles

CO2 as Feedback



Pam Martin, University of Chicago, 2010

Glacial Cycles

Greenhouse Gas Feedback Model

$$c\frac{dT}{dt} = S(t) + G(C) - \sigma T^4,$$
$$\frac{dC}{dt} = V - (W_0 + W_1 C) + \beta(C_{\max} - C)\max\left(\frac{dT}{dt} - \varepsilon, 0\right).$$

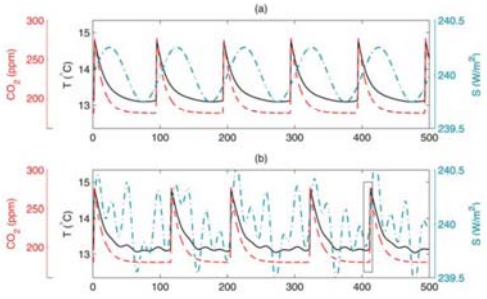
volcanos; weathering; CO2 outgassing

$$S(t) = \bar{S} + \sum_i S_i \sin\left(\frac{2\pi t}{\Gamma_i}\right) \quad \text{insolation}$$
$$G(C) = \bar{G} + A\ln\left(\frac{C}{C_0}\right) \quad \text{greenhouse forcing}$$

Andrew Hogg, *Geophysical Research Letters* **35** (2008)

Glacial Cycles

Greenhouse Gas Feedback Model



Andrew Hogg, *Geophysical Research Letters* **35** (2008)

Glacial Cycles

Questions

1. Did eccentricity play any role during the last million years, or is the apparent 100 Kyr cycle an artifact (Huybers).

2. Is the CO_2 feedback sufficient to explain the increasing amplitude and period of the glacial cycles during the last million years, *i.e.*, is it the mechanism behind the Huyber model.

3. Where does the atmospheric CO_2 go during the glacial maxima?
The ocean? The land?

4. What will be the effect of the anthropogenic CO_2?