Budyko's Energy Balance Model: To an Infinite Dimensional Space and Beyond

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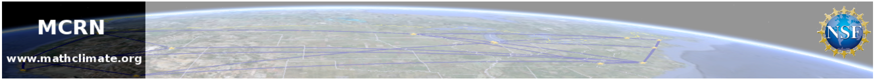
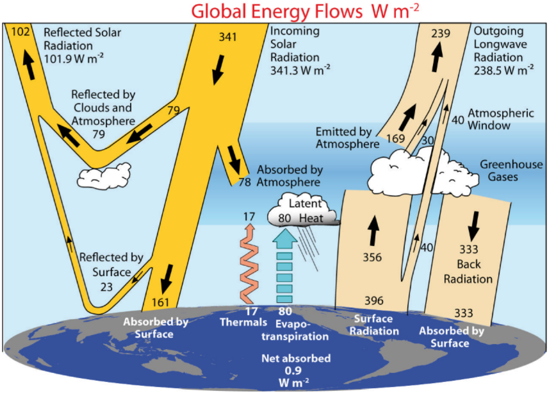


Summary of today's talk

- Background: energy balance models (EBM), Budyko's EBM, ice line equation
- An infinite dimensional version of Budyko's EBM, $1 - D$ invariant manifold, and some examples, eg *Jormundgand* world
- Opportunities: greenhouse gas feedback, snowball, piecewise smooth differential equations, maps

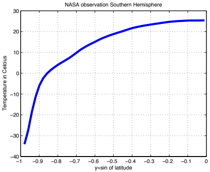
What does Earth do with all that energy from the Sun?

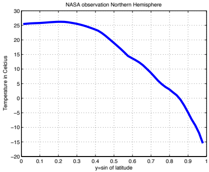


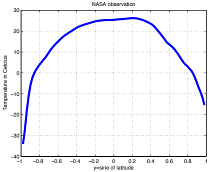
Earth's temperature profiles

NASA's observation

Southern Hemisphere **Northern Hemisphere**







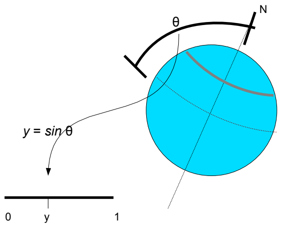
MCRN
www.mathclimate.org

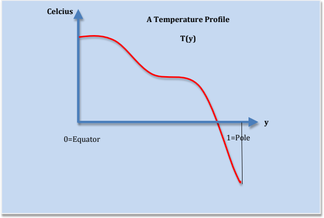
Source: Emma Cutler, Bowdoin College, data from http://www.giss.nasa.gov/ar5/lplat.html

NSF

Simplifying the work: symmetric temperature profile

1. Symmetry about the equator, so we only look at eg. the northern hemisphere.
2. Annual average along the same latitude, say θ.

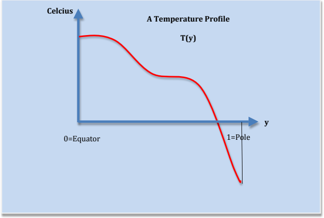




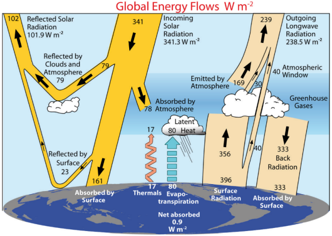
Energy balance principle

Incoming Solar Radiation (Insolation) = Reflected Energy + Outgoing Longwave Radiation + Transported Energy

What we want to model



What we have



Energy balance principle

Changes in energy or energy imbalance

=

(Heat capacity) · (Temperature change ΔT)

=

(Insolation energy absorbed after albedo effect)
- (Radiated energy/ OLR) + (Transported energy)

Budyko's EBM

$$R\Delta[T(y)](t) = k\left[\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{insolation after albedo effect}} - \underbrace{(A+B\cdot T(y))}_{\text{re-emission/ OLR}} + \underbrace{C\cdot(\overline{T}-T(y))}_{\text{transported energy}}\right]$$

$$T = T(y) = T(t,y)$$

annually and latitudinally averaged temperature profile

$$\Delta T(t,y) = T(t+1,y) - T(t,y)$$

(with the right k, unit time = year)

Budyko's EBM: The OLR term and the transport term

$$R\Delta[T(y)](t) = k\left[\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{insolation after albedo effect}} - \underbrace{(A+B\cdot T)}_{\text{re-emission/ OLR}} + \underbrace{C\cdot(\overline{T}-T)}_{\text{transported energy}}\right]$$

$-(A+BT)$: The outgoing long wave radiation is a linearized version of the *Stephan-Boltzman*'s law σT^4

$C(\overline{T}-T)$: The transport term assumes that the temperature at y decays to the global temperature.

$$A \cong 202 watts\, m^{-2} \quad B \cong 1.9 watts\, m^{-2} C^{-1}\; C \cong 1.6B$$

(K. K. Tung, 2007)

Budyko's EBM: insolation after albedo effect

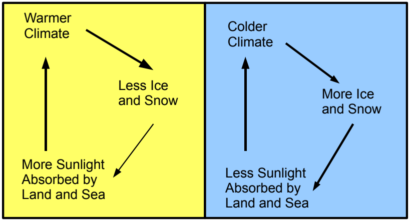
$$R\Delta[T(y)](t) = k\left[\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{insolation after albedo effect}} - \underbrace{(A + B\cdot T)}_{\text{re-emission/ OLR}} + \underbrace{C\cdot(\overline{T} - T)}_{\text{transported energy}}\right]$$

$$Q \cdot s(y) \cdot (1 - \alpha(\eta, y))$$

$Q =$ the solar constant $\cong 341 watts\, m^{-2}$
$s(y)$ is a distribution function, 2nd degree Legendre approximation
$s(y) = 0.482\frac{3y^2-1}{2}$
$\alpha(\eta, y) =$ the albedo at y given that the iceline is at η, here, it is chosen to be smooth and bounded both in y and in η.

The ice albedo has a positive feedback

$\alpha(\eta, y) =$ the albedo at y given that the ice line is at η



The Ice Albedo Feedback

The ice albedo has a positive feedback

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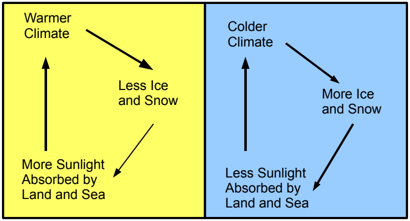


The Ice Albedo Feedback

Need some ice line dynamics.

Ice line dynamics

How should ice line evolve?

Ice forms (or melts) slowly when the temperature falls (or rises) below a certain critical temperature T_c

$$\Delta[\eta](t) = \eta(t+1) - \eta(t) = \varepsilon \left(T(\eta(t)) - T_c \right)$$

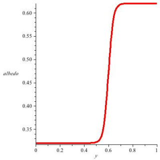
Here, we assume ε is small, though others might disagree.

Animations

Starting temperature profile $T(y) = 34y^2 - 54$, with a smooth albedo function.

$$\alpha(\eta)(y) = 0.47 + 0.15 \cdot [tanh(M(y - \eta))]$$

$$\eta = 0.6 \text{ and } M = 25$$



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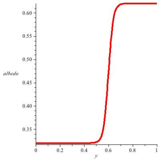


Is there an invariant set? Is it attracting? What is the function space?

Budyko's time one map m

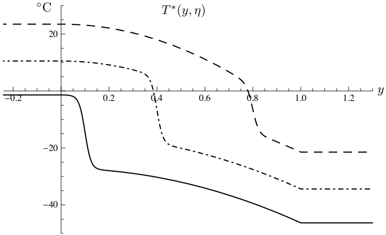
$$\begin{cases} \Delta[T(y)](t) & = F([T(y),\eta])(t), \qquad y \in (0,1) \\ & = Qs(y)[1-\alpha(\eta,y)] - [A + BT(y)] + C[\overline{T} - T(y)] \\ \\ \Delta[\eta](t) & = G([T(y),\eta])(t) \\ & = \varepsilon\,(T(\eta) - T_c) \end{cases}$$

The time one map m associated with the Budyko-ice line system:

$$m[T(y),\eta](t+1) = [T(y),\eta](t) + \cdot\Delta\left([T(y),\eta]\right)(t) \qquad (1)$$

$T(y)$ is a bounded continuous function with the sup norm over $\mathbb{R}$ and $\eta \in \mathbb{R}$

The critical set $\mathcal{T}$



$$\mathcal{T} := \{T^*(\eta, y) : F(T^*(\eta, y), \eta) = 0\}$$

$$F([T(y), \eta])(t) = Qs(y)[1 - \alpha(\eta, y)] - [A + BT(y)] + C[\overline{T} - T(y)]$$

An attracting invariant manifold result

Theorem

Under some parameter conditions, when ϵ is sufficiently small, **there exists an attracting one dimensional invariant manifold** *for the time one map m associated with the Budyko's equation. (W-, 2010)*

Corollary

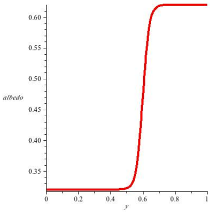
The invariant manifold is within $O(\epsilon)$ of the critical set $\mathcal{T}^*$

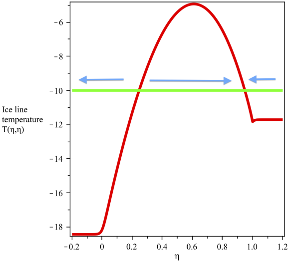
We call this 1-D invariant manifold Φ^*.

Example 1: $\alpha(\eta, y)$ as in the animations

Recall, $\Delta[\eta] = \varepsilon(T(\eta) - T_c)$

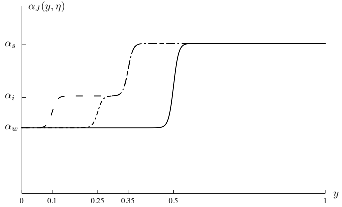
The equilibrium ice line temperature $T^*(\eta, \eta)$



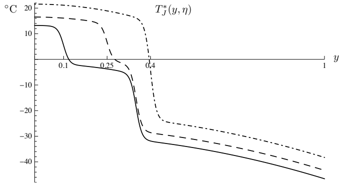


Example 2: *Jormungand* world

The albedo function



Some equilibrium temperature profiles

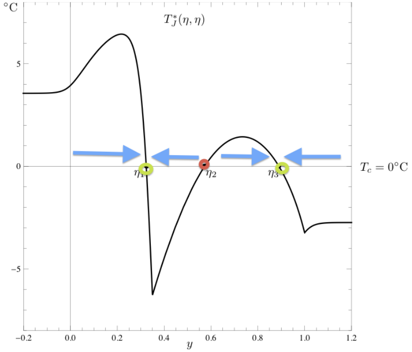


(Walsh and W-, 2012)

Jormungand state: *the ocean is very nearly globally ice covered, but a very small strip of the tropical ocean remains ice-free.* Abbot, et al, 2011.

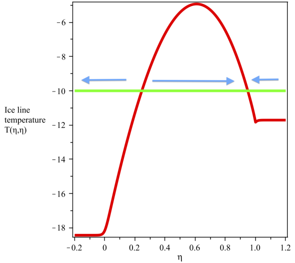
Example 2: *Jormungand* world

The ice line dynamics



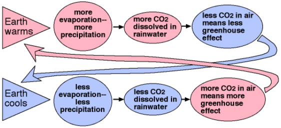
From snowball to ice free state

Lower latitude continents allowed for an albedo runaway snowball
(ie. via a saddle node bifurcation).



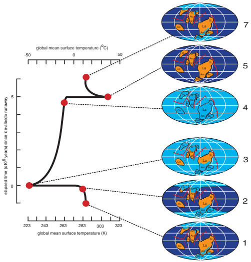
vink, 1992). On a snowball Earth, volcanoes would continue to pump CO_2 into the atmosphere (and ocean), but the sinks for CO_2 – silicate weathering and photosynthesis – would be largely eliminated (Kirschvink, 1992).

Hoffman & Schrag, *Snowball Earth Hypothesis*,2002



Carbon cycle is the Earth's thermostat

Beyond albedo: the greenhouse gas feedback



Back to Budyko's EBM:

$$\Delta[T(y)] = Qs(y)[1-\alpha(\eta, y)]-[A+BT(y)]+C[\overline{T}-T(y)]$$

The parameter A needs to be dynamically driven, one guess:

$$\Delta A = \delta(\eta - \eta_c)$$

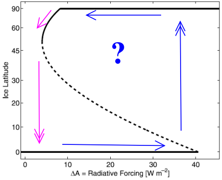
Beyond albedo: the greenhouse gas feedback

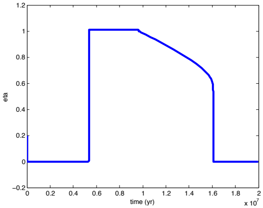
Use the invariant manifold $\Phi^*(\eta, y)$ of the Budyko-ice line system for η

$$A' = \delta(\eta - \eta_c)$$

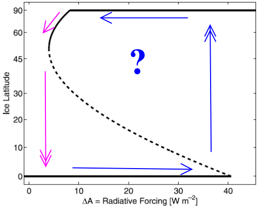
$$\eta' = \epsilon(\Phi^*(\eta, \eta) - T_c)$$

Here $\Delta A = A_0 - A$





Beyond albedo: the greenhouse gas feedback



Challenges: what happen at the boundaries, ie. $\eta = 0, 1$?
Are there machineries eg. maps, piecewise smooth system?

Thank you for your attentions!!