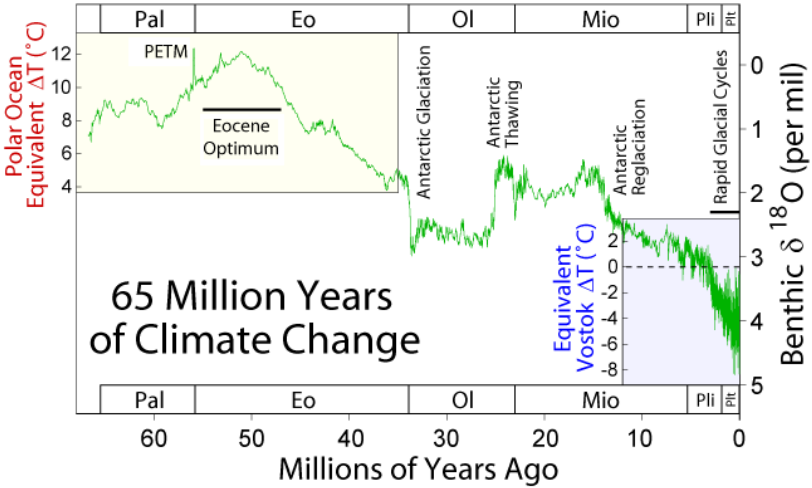
The Compost-Bomb Instability and the PETM

Jonathan Hahn

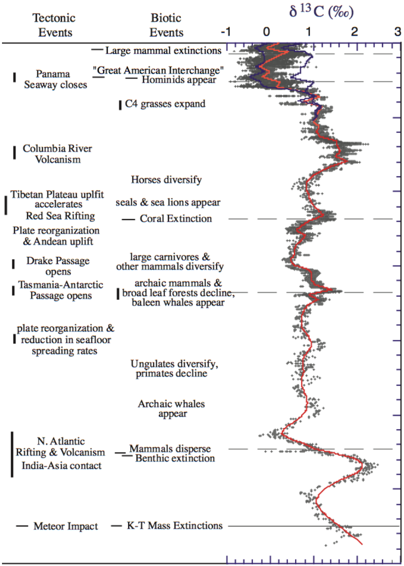
UMN Mathematics of Climate Seminar

November 12, 2013

The Paleocene-Eocene Thermal Maximum



The Paleocene-Eocene Thermal Maximum



Carbon Mass Balance

$$M_{final} \times \delta^{13}C_{final} = M_{initial} \times \delta^{13}C_{initial} + M_{added} \times \delta^{13}C_{added}$$

$$M_{final} = M_{initial} + M_{added}$$

$$CIE = \delta^{13}C_{final} - \delta^{13}C_{initial}$$

$$M_{added} = -CIE \times \frac{M_{initial}}{\delta^{13}C_{final} - \delta^{13}C_{added}}$$

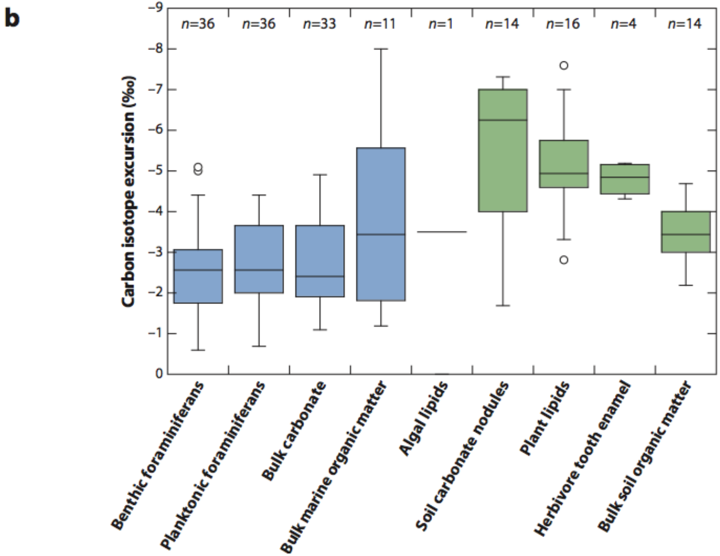
$$\delta^{13}C_{sample} = (R_{sample}/R_{standard} - 1)$$

$$R_{sample} = C^{13}/C^{12}$$

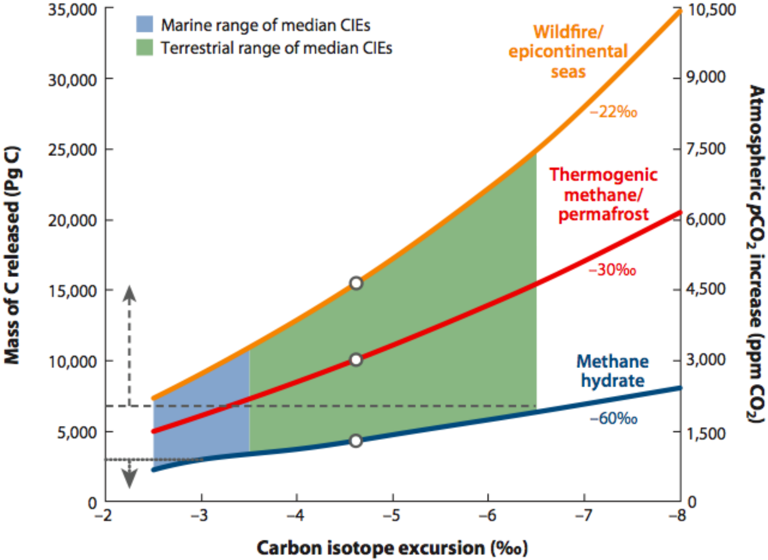
$\delta^{13}C$ of Carbon Sources

Source	$\delta^{13}C$
Methane clathrates:	-6%
Wildfires:	-2.2%
Drying of inland seas:	-2.2%
Thermogenic methane:	-3%
Permafrost and peat:	-3%

Estimating the CIE



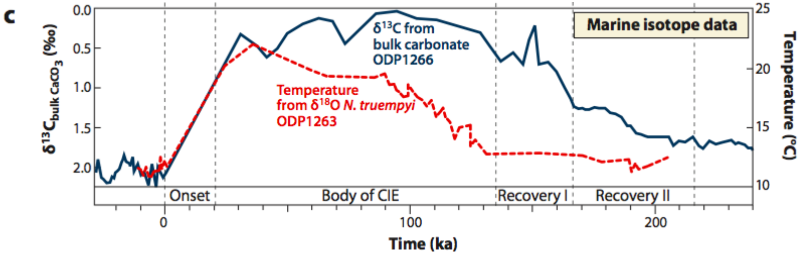
Mass of Carbon Added to the Atmosphere



Source of carbon unclear

Zeebe et. al.:	must have been methane
Panchuck et. al. and Pagani et. al.:	definitely not methane
Wright & Schaller:	probably a comet
Most geologists:	unlikely to be a comet

CIE time frame

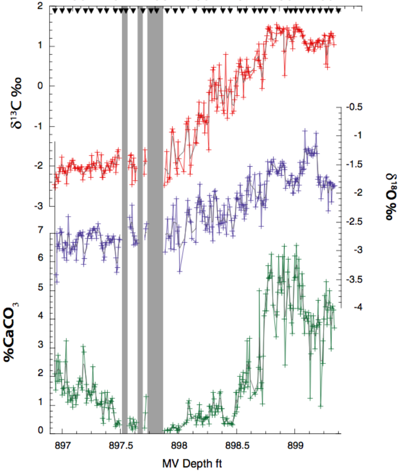


This data suggests the onset happens in about 20,000 years

CIE time frame

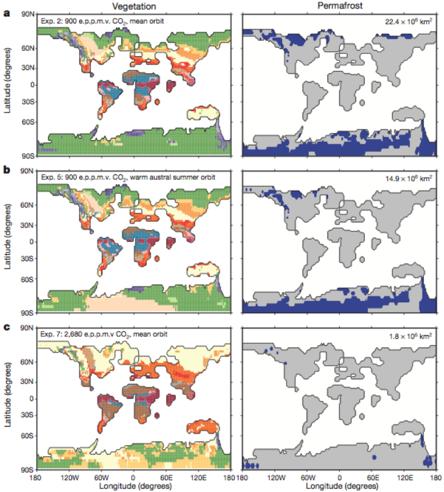
Wright and Schaller say:

13 years



Peat and Permafrost

DeConto et. al. use GCM with orbital forcing



Compost-Bomb Instability

Luke & Cox:

The compost bomb: thermal instability in peatland soils

$$\frac{dC}{dt} = \Pi - Cr(T)$$

$$\epsilon \frac{dT}{dt} = Cr(T) - \frac{\lambda}{A}(T - T_a)$$

C	Vertically integrated soil carbon content (kg/m^2)
T	Soil temperature
T_a	Atmospheric Temperature
$r(T) = r_0 e^{\alpha T}$	Soil respiration rate

Compost-Bomb Instability

$$\frac{dC}{dt} = \Pi - Cr(T)$$

$$\epsilon \frac{dT}{dt} = Cr(T) - \frac{\lambda}{A}(T - T_a)$$

$$r(T) = r_0 e^{\alpha T}$$

$r_0 = 0.01(yr^{-1})$; $\alpha = ln(2.5)/10$	microbial respiration parameters
$Pi = 1.055(kgm^{-2}yr^{-1})$	litter fall from plants
$\lambda = 5.049 * 10^6$ $(Jyr^{-1}m^{-2} \deg C^{-1})$	soil-to-air heat transfer coefficient
$A = 3.9 * 10^7(Jkg^{-1})$	specific heat from microbial respiration
$\mu = 2.5 * 10^6(Jm^{-2} \deg C^{-1})$	soil heat capacity
$\epsilon = \mu/A = .064$	

Compost-Bomb Instability

Equilibrium analysis

$$0 = \frac{dC}{dt} = \Pi - C^{eq} r(T^{eq})$$

$$0 = \frac{dT}{dt} = C^{eq} r(T^{eq}) - \frac{\lambda}{A}(T^{eq} - T_a)$$

Compost-Bomb Instability

Equilibrium analysis

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$$C^{eq} = \Pi / r(T^{eq})$$

$$T^{eq} = A C^{eq} r(T^{eq}) / \lambda + T_a$$

Compost-Bomb Instability

Equilibrium analysis

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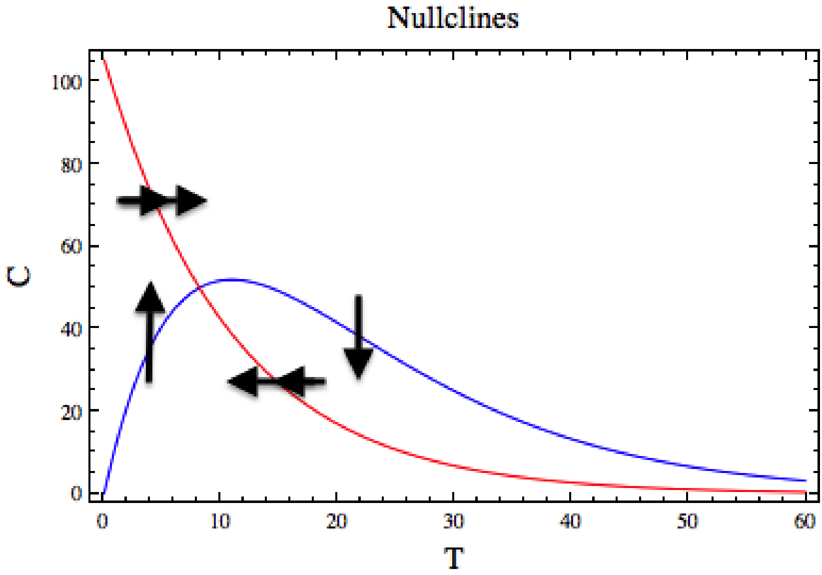
$$C^{eq} = \Pi / r(T^{eq})$$
$$T^{eq} = A\Pi/\lambda + T_a$$

Compost-Bomb Instability



Compost-Bomb Instability

$$J = \begin{pmatrix} -\frac{\lambda}{A\epsilon} + \frac{C\alpha r(T)}{\epsilon} & \frac{r(T)}{\epsilon} \\ \\ -C\alpha r(T) & -r(T) \end{pmatrix}$$

$$\det(J - I\gamma) = \gamma^2 + \gamma\left(\frac{\lambda}{A\epsilon} - \frac{C\alpha r(T)}{\epsilon} + r(T)\right) + \frac{\lambda}{A\epsilon} r(T)$$

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Instability condition: $\frac{\lambda}{A\epsilon} - \frac{C^{eq}\alpha r(T^{eq})}{\epsilon} + r(T^{eq}) < 0$

$$\frac{\lambda}{A\epsilon} - \frac{\Pi\alpha}{\epsilon} + r(T^{eq}) < 0$$

Compost-Bomb Instability

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$$\frac{\lambda}{A\epsilon} - \frac{\Pi\alpha}{\epsilon} + r(T^{eq}) < 0$$

Stable!

Compost-Bomb Instability

However, Luke & Cox claim there is instability if the decrease in soil carbon cannot keep up with the increase in soil temperature.

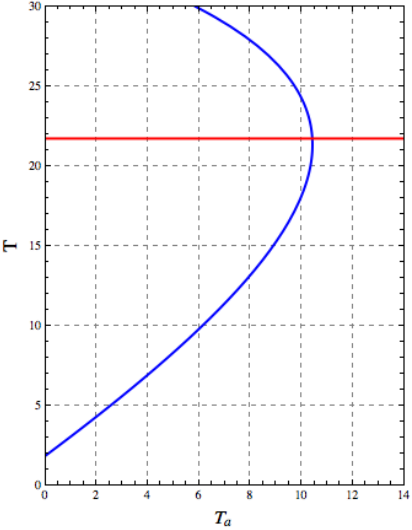
Assume $C = C_0$ is constant ($\frac{dC}{dt} << \frac{dT}{dt}$).
Then according to the previous condition, the system is unstable if:

$$\frac{\lambda}{A\epsilon} - \frac{C_0 \alpha r(T^{eq})}{\epsilon} + r(T^{eq}) < 0$$

$$T^{eq} > (1/\alpha) \ln \left(\frac{-\lambda/(r_0 A\epsilon)}{1 - C_0\alpha/\epsilon} \right)$$

Compost-Bomb Instability

Equilibrium for T are given by: $T^{eq} - AC_0 r(T^{eq})/\lambda = T_a$



Canard Trajectories

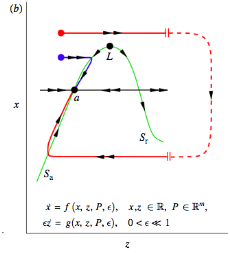
Wieczorek et. al. examine the same system but including:

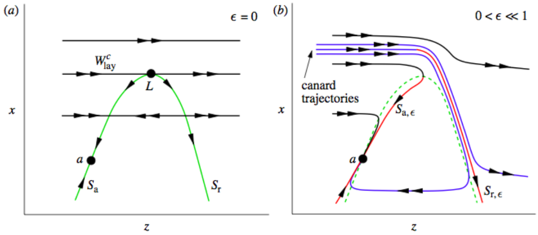
$$\frac{dT_a}{dt} = v$$

In this system, a "canard explosion" occurs for $v > v_{critical}$.

Canards: trajectories which transition from small amplitude limit cycles to large ones with a small perturbation in a parameter.

Canard Trajectories





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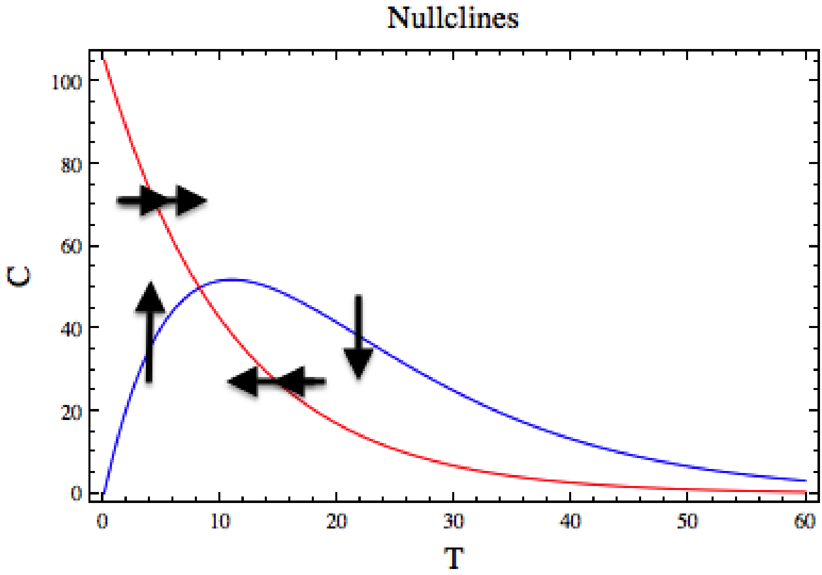

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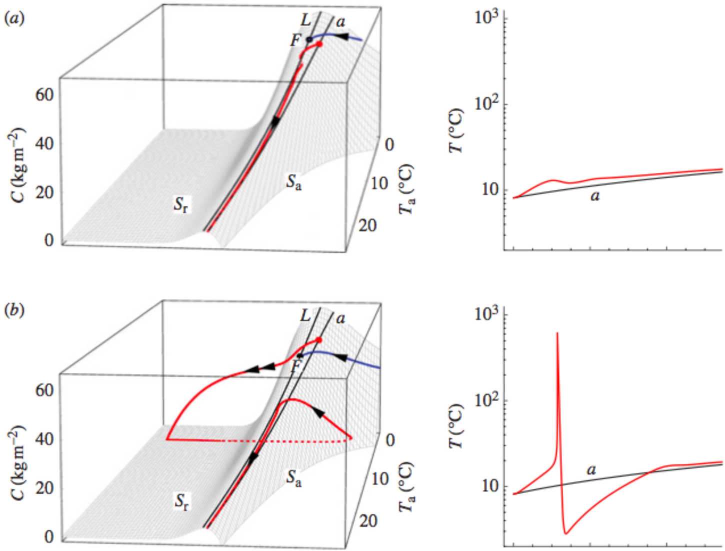

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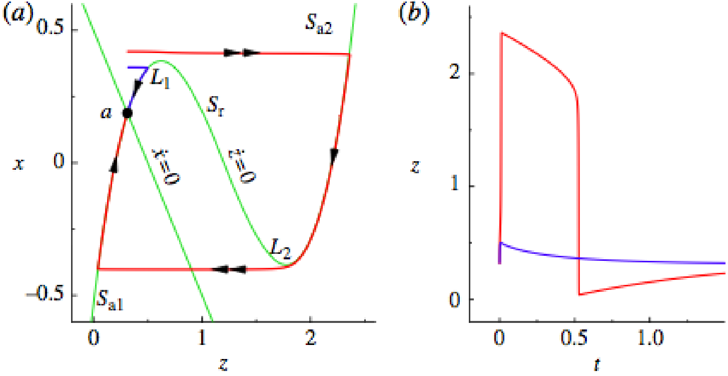

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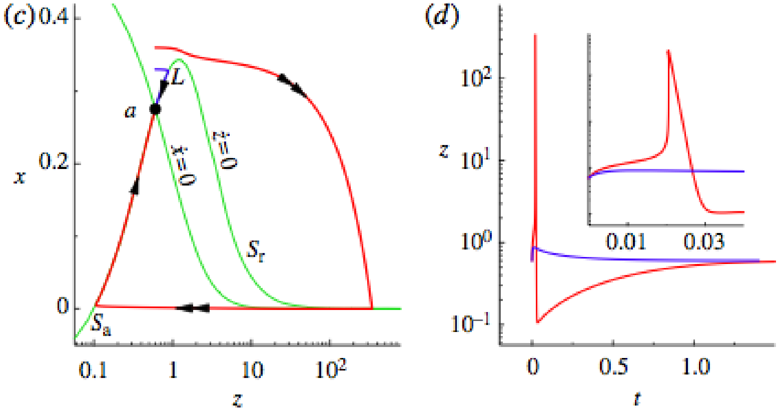

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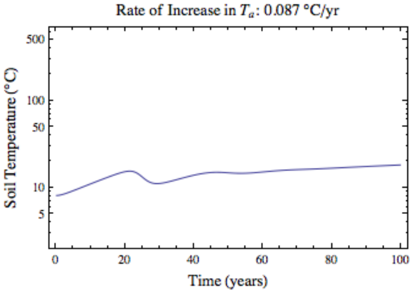

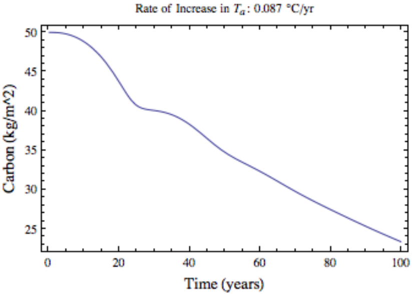

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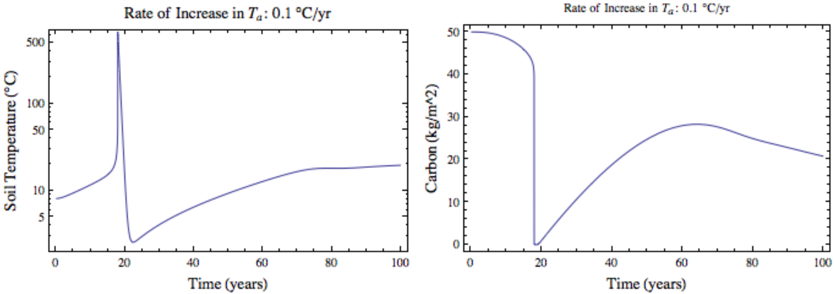

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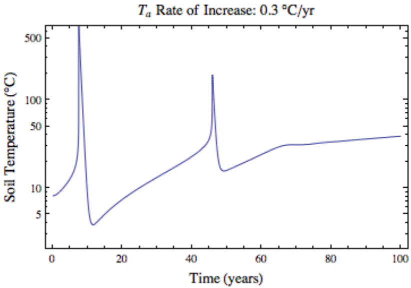

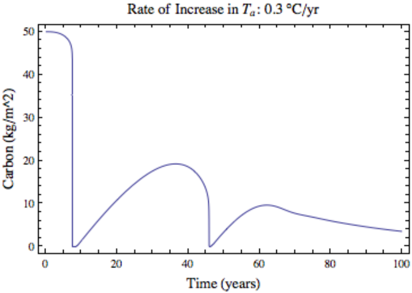

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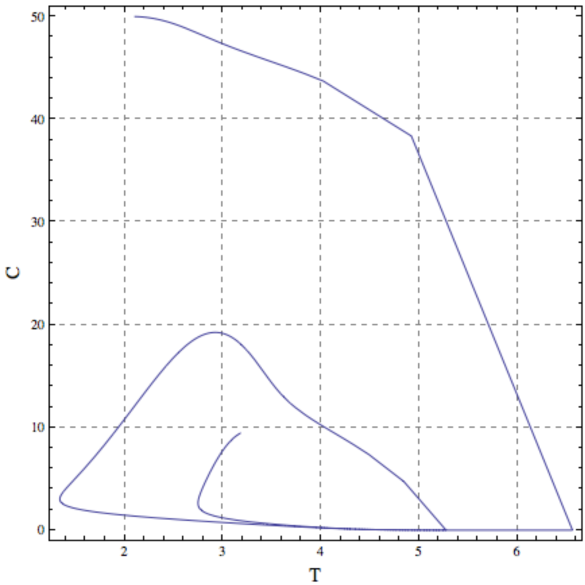

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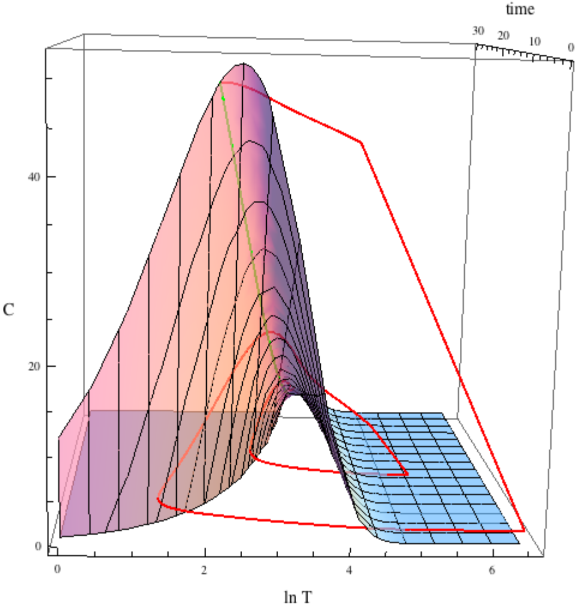

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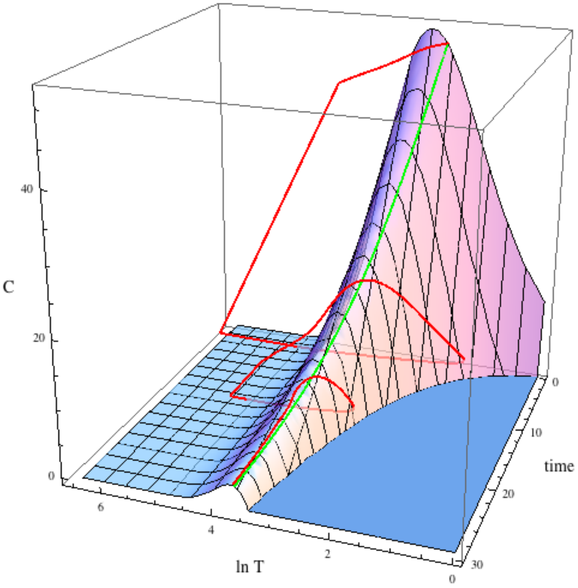

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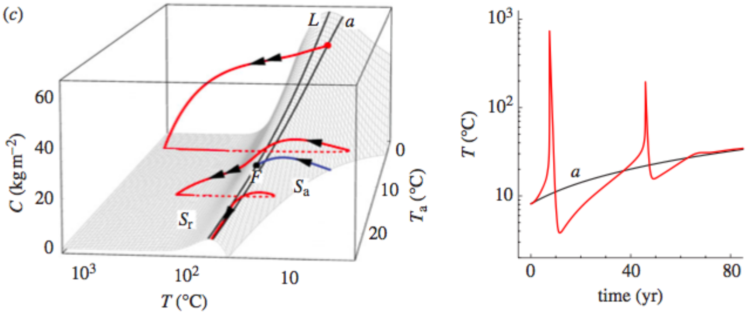

Rate-induced tipping

Credits

McInerny & Wing: “The Paleocene-Eocene Thermal Maximum: A Perturbation of Carbon Cycle, Climate, and Biosphere with Implications for the Future”

Luke, C. M, & Cox, P.M.: “Soil carbon and climate change: from the Jenkinson effect to the compost-bomb instability”

Wieczorek et. al.: “Excitability in ramped systems: the compost-bomb instability”