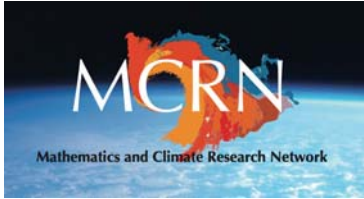


The Earth's Energy Imbalance
 Richard McGehee
 School of Mathematics
 University of Minnesota
 Mathematics of Climate Seminar
 October 6, 2015



Energy Imbalance



Conservation of Energy

temperature change ~ energy in - energy out

short wave energy
from the Sun

long wave energy
from the Earth

Everything else is detail.

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Energy Imbalance



Stefan-Boltzmann Law

power flux (W/m²)

$F = \sigma T^4$

temperature (K)

Stefan-Boltzmann constant
 $\sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

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Energy Imbalance

Stefan-Boltzmann Law

power flux (W/m²)



$F = \sigma T^4$

temperature (K)


Stefan-Boltzmann constant
 $\sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

Example

surface temperature of the Sun: 5780K
 power flux: $5.67 \times 10^{-8} \times (5780)^4 = 6.33 \times 10^7 \text{ W/m}^2$
 total solar power output: $6.33 \times 10^7 \times 4\pi(r_s)^2$,
 where r_s = radius of the sun = $6.96 \times 10^8 \text{ m}$
 total solar output: $3.85 \times 10^{26} \text{ W}$
 200 nanoseconds = time it takes for the sun to produce
 the equivalent of the annual global electricity
 production ($7.3 \times 10^{19} \text{ Joules}$)

<http://astronomybythecosmos.com/tag/sun/>



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Energy Imbalance

Insolation



Solar flux at a distance r from the sun:

$$F = \frac{6.33 \times 10^7 4\pi r_s^2}{4\pi r^2} = 6.33 \times 10^7 \left(\frac{r_s}{r}\right)^2 \text{ W/m}^2$$

$r_s = 6.96 \times 10^8 \text{ m}$
 $r = 1.5 \times 10^{11} \text{ m}$
 $F = 1368 \text{ W/m}^2$

Power intercepted by the Earth:

$F \times \pi r_E^2$ Watts, r_E = radius of Earth = $6.37 \times 10^6 \text{ m}$
 $F = 1.74 \times 10^{17} \text{ W}$

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Energy Imbalance

Insolation

Solar flux at a distance r from the sun:




$$F = \frac{6.33 \times 10^7 4\pi r_s^2}{4\pi r^2} = 6.33 \times 10^7 \left(\frac{r_s}{r}\right)^2 \text{ W/m}^2$$

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 $F = 1.74 \times 10^{17} \text{ W}$

Biologically Stored Energy
 total coal reserves: 10^{15} kg
 energy content: $3 \times 10^7 \text{ J/kg}$
 total energy in coal reserves: $3 \times 10^{22} \text{ J}$
 = 2 days of insolation

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Energy Imbalance

Insolation

Global Average Insolation intercepted flux: $F = 1368 \text{ W/m}^2$
 Earth cross-section: πr_E^2
 surface area: $4\pi r_E^2$
 average flux: $1368/4 = 342 \text{ W/m}^2 = Q$

Simple Model
 Assume that Earth is a perfectly thermally conducting black body.

$$Q = \sigma T^4$$

$$T = (Q/\sigma)^{1/4} = (342/5.67 \times 10^{-8})^{1/4}$$

$$= \boxed{279\text{K} = 6\text{ C} = 43\text{ F}}$$

Dynamics

$$R \frac{dT}{dt} = Q - \sigma T^4$$

heat capacity \rightarrow R \leftarrow stable equilibrium

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Energy Imbalance

Albedo

Not all the insolation reaches the surface. Some is reflected back into space.
 The proportion reflected is called the albedo, denoted α .
 For Earth, $\alpha \approx 0.3$.

Simple Model
 Assume that Earth is a perfectly thermally conducting black body, but only 70% of the insolation is absorbed.

$$T = (0.7 \cdot F/\sigma)^{1/4} = (0.7 \cdot 342/5.67 \times 10^{-8})^{1/4}$$

$$= \boxed{255\text{K} = -18\text{ C} = 0\text{ F}}$$

Dynamics

$$R \frac{dT}{dt} = Q(1 - \alpha) - \sigma T^4$$

\leftarrow stable equilibrium

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Energy Imbalance

Albedo


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Why isn't the Earth a snowball?



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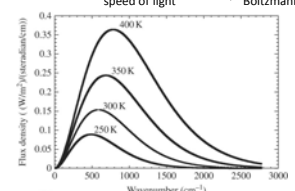
Energy Imbalance

Black Body Radiation

Planck's Function

$$\text{flux density} \rightarrow B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

frequency $\rightarrow \nu$ \leftarrow Planck's constant
 temperature $\rightarrow T$ \leftarrow Boltzmann's constant
 c \leftarrow speed of light

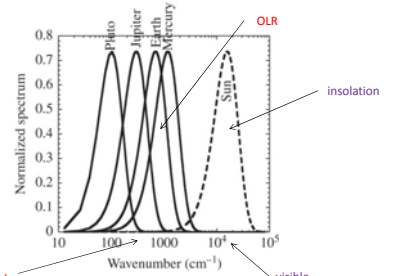


Raymond T. Pierrehumbert, *Principles of Planetary Climate*, Cambridge University Press, 2010.
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Energy Imbalance

Insolation vs. OLR

OLR = Outgoing Longwave Radiation



Normalized spectrum vs. Wavenumber (cm^{-1})

Planets shown: Pluto, Jupiter, Earth, Mercury. Sun's spectrum is also shown.

insolation (dashed line) vs. OLR (solid lines)

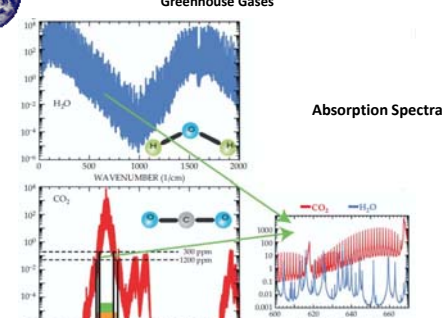
infrared vs. visible regions are indicated.

Raymond T. Pierrehumbert, *Principles of Planetary Climate*, Cambridge University Press, 2010.
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Energy Imbalance

Greenhouse Gases

Absorption Spectra

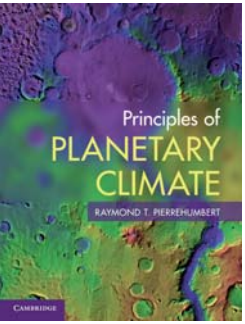


Top graph: H_2O absorption spectrum (Wavenumber 500-2000 cm^{-1})

Bottom graph: CO_2 absorption spectrum (Wavenumber 500-2000 cm^{-1})

Bottom-right inset: CO_2 and H_2O absorption spectra (Wavenumber 600-1600 cm^{-1})


Raymond T. Pierrehumbert, *Infrared radiation and planetary temperature*, *Physics Today*, 2011.
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Energy Imbalance
Greenhouse Effect

How to learn more.

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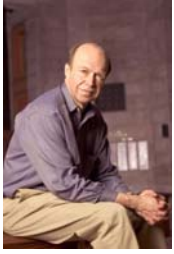


Energy Imbalance

James Hansen, et al, *Earth's Energy Imbalance: Confirmation and Implications*, SCIENCE 308 (2005), p. 1431


Average Insolation: 342 Watts per square meter
Current Heat Imbalance:
0.85 ± 0.15 Wm²

Unit of energy: Watt-year (W yr)
1 W yr = 3.15x10⁷ J
= 8761 W hr = 8.761 kWh
Yearly Insolation:
342 W yr m² = 1.08 x 10¹⁰ J



https://en.wikipedia.org/wiki/James_Hansen

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Energy Imbalance

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Ice sheet melting to raise sea level 1 meter. Assume ice starts at -10°C and ends at mean ocean surface temperature (+15°C). Energy required is 100 cal/g (80 cal/g for melting). Energy for 1 meter of sea level: 100g/cm² × 100cal/g = 4.19 joules/cal = area Earth × 0.7 = 1.5 × 10²³ joules = 9.3 W yr m².

Sea ice melting (all sea ice on planet). Assume ice starts at -10°C and ends at mean ocean surface temperature (+15°C), and that sea ice covers 4% of the planet with mean thickness 2.5 m. Energy required is 250 g/cm² × 100 cal/g (80 cal/g for melting) = 4.19 joules/cal × 0.04 = area Earth × 2.14 × 10²² joules = 1.3 W yr m².


Air warming by 1°C. The Earth's atmospheric mass is ~ 10¹⁸ m of water. Heat capacity of air = 0.24 cal/g°C. Energy to raise air temperature 1°C: 1°C × 1000 g/cm³ × 0.24 cal/g°C × 4.19 joules/cal = area Earth × 0.26 × 10²³ joules = 0.32 W yr m².

Land surface warming by 1°C. The depth of penetration of a thermal wave into the Earth's crust in 10 years, weighted by ΔT, is ~ 10 m. With density ~ 3 g/cm³, heat capacity ~ 0.2 cal/g°C, and 0.29 fractional land coverage, land heat storage is 10³ cm × 3 g/cm³ × 0.2 cal/g°C × 1°C × 4.19 joules/cal = area Earth × 0.29 × 0.37 × 10²³ joules = 0.23 W yr. [In a century the depth of penetration is ~3 times more than in a decade, so heat storage in a century due to 1°C warming is ~ 0.7 W yr m².]

¹Note that 1 W sec = 1 joule, # sec/year = π × 10⁷, area Earth = 5.1 × 10¹⁸ cm², 1 W yr over full Earth = 1.61 × 10²³ joules, ocean fraction of Earth = 0.7, 1 calorie = 4.19 joules.

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Energy Imbalance


Assume all insolation goes toward warming.

Energy to warm air 1°C: 0.32 W yr m²
Power from Sun: 342 Wm²
Time: 0.32/342 = 0.00094 yr = **8.2 hr**

Assume a global heat imbalance of 0.85 W m².

Energy to warm air 1°C: 0.32 W yr m²
Heat imbalance: 0.85 Wm²
Time: 0.32/0.85 = 0.376 yr = **4.5 months**

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Energy Imbalance

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Energy Imbalance


Assume all insolation goes toward warming.

Energy to warm land surface 1°C: 0.7 W yr m²
Power from Sun: 342 Wm²
Time: 0.7/342 = 0.0020 yr = **18 hr**

Assume a global heat imbalance of 0.85 W m².

Energy to warm land surface 1°C: 0.7 W yr m²
Heat imbalance: 0.85 Wm²
Time: 0.7/0.85 = 0.376 yr = **9.9 months**

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Energy Imbalance

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Energy Imbalance


Assume all insolation goes toward warming.

Energy to melt all the sea ice: 1.3 W yr m^2
Power from Sun: 342 W m^2
Time: $1.3/342 = 0.0038 \text{ yr} = \mathbf{33 \text{ hr}}$

Assume a global heat imbalance of 0.85 W m^2 .

Energy to melt all the sea ice: 1.3 W yr m^2
Heat imbalance: 0.85 W m^2
Time: $1.3/0.85 = 1.53 \text{ yr} = \mathbf{18 \text{ months}}$

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Energy Imbalance


Assume all insolation goes toward warming.

Energy to raise sea level 1 meter by melting ice sheets: 9.3 W yr m^2
Power from Sun: 342 W m^2
Time: $9.3/342 = 0.027 \text{ yr} = \mathbf{9.9 \text{ days}}$

Assume a global heat imbalance of 0.85 W m^2 .

Energy to raise sea level 1 meter by melting ice sheets: 9.3 W yr m^2
Heat imbalance: 0.85 W m^2
Time: $9.3/0.85 = \mathbf{10.9 \text{ yr}}$

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Energy Imbalance


Assume all insolation goes toward warming.

Energy to warm ocean 1°C to depth 1 km: 93 W yr m^2
Power from Sun: 342 W m^2
Time: $93/342 = 0.27 \text{ yr} = \mathbf{3.3 \text{ months}}$

Assume a global heat imbalance of 0.85 W m^2 .

Energy to warm ocean 1°C to depth 1 km: 93 W yr m^2
Heat imbalance: 0.85 W m^2
Time: $93/0.85 = \mathbf{109 \text{ years}}$

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Energy Imbalance

What if we melt all the ice sheets?


All ice sheets will raise the sea level 70 m.
 Will take $9.3 \text{ W yr m}^{-2} \times 70 = 650 \text{ W yr m}^{-2}$

Assume all insolation goes toward warming.


Energy to melting all ice sheets: 650 W yr m^{-2}
 Power from Sun: 342 W m^{-2}
 Time: $650/342 = 1.9 \text{ years}$

Assume a global heat imbalance of 0.85 W m^{-2} .

Energy to melting all ice sheets: 650 W yr m^{-2}
 Heat imbalance: 0.85 W m^{-2}
 Time: $650/0.85 = 765 \text{ years}$



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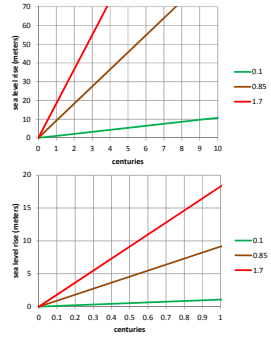



Energy Imbalance


Suppose that all the heat imbalance went to melting the glaciers.

It takes 9.3 W yr m^{-2} to turn glaciers into 1 meter of ocean. If the heat imbalance is $w \text{ W m}^{-2}$, the sea level would rise at the rate of $w/9.3$ meters per year. At the current imbalance of 0.85 W m^{-2} , the rate is about 0.109 meters per year, or 10.9 meters per century.

Melting all the glaciers would cause a sea level rise of about 70 meters and would take about 765 years at the current imbalance.

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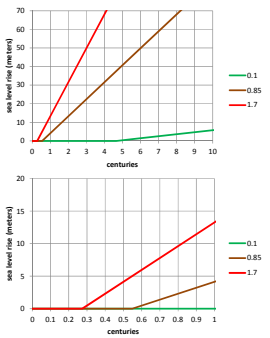



Energy Imbalance


Suppose now that all the heat imbalance first goes to raising the top kilometer of ocean by 0.5°C , and then goes to melting the glaciers.

It takes 46.5 W yr m^{-2} to raise the temperature of a kilometer of ocean by 0.5°C . If the heat imbalance is $w \text{ W m}^{-2}$, the increase would be achieved in $46.5/w$ years, after which the sea level would rise at $w/9.3$ meters per year.

At the current imbalance of 0.85 W m^{-2} , the ocean temperature increase would delay the sea level rise by about 55 years.

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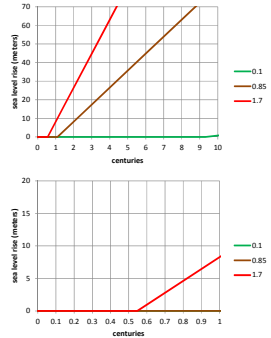



Energy Imbalance


Suppose instead that all the heat imbalance first goes to raising the top kilometer of ocean by 1°C , and then goes to melting the glaciers.

It takes 93 W yr m^{-2} to raise the temperature of a kilometer of ocean by 1°C . If the heat imbalance is $w \text{ W m}^{-2}$, the increase would be achieved in $93/w$ years, after which the sea level would rise at $w/9.3$ meters per year.

At the current imbalance of 0.85 W m^{-2} , the ocean temperature increase would delay the sea level rise by about 109 years.

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Energy Imbalance


Summary

Currently, it appears that the heat imbalance is mostly going to heating the ocean, not to melting ice. If this pattern continues, the danger for this century is more likely to come from weather changes than from sea level rise.

The current heat imbalance has the potential to raise the sea level by almost a meter per decade, a major threat to coastal cities worldwide.




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Energy Imbalance

Questions about the Coming Centuries

- How will the heat imbalance be divided between heating the ocean and melting the glaciers?
- How will the heat imbalance be affected by increasing atmospheric greenhouse gases?
- How will the heat imbalance be affected by increasing ocean temperatures?
- What happens to the weather as the ocean temperature rises and the ice caps melt?
- What should we do about coastal cities?



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Energy Imbalance


What can mathematicians do?

Models
Minimal complexity, aka "conceptual", "simple", "toy"
Intermediate complexity
Maximal complexity, aka "GCM"

Data
Parameter estimation (statistics)
Data assimilation

Quantification
Uncertainty
Resilience

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