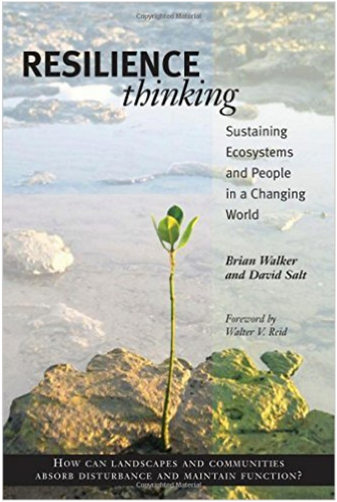
Prospects for Resilience Quantification Using Dynamical Systems Theory

Kate Meyer
Minnesota Math Climate Seminar
March 22nd, 2016

Outline

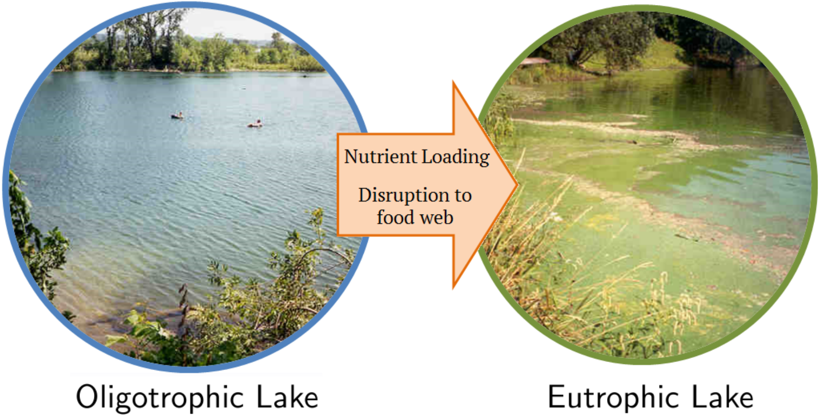


Resilience

"[T]he capacity of [a] system to absorb change and disturbances and still retain its basic structure and function"

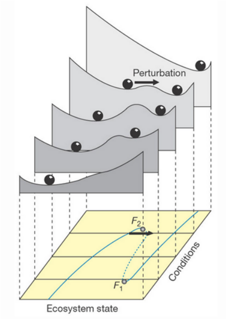
– Walker & Salt (2006)

Example



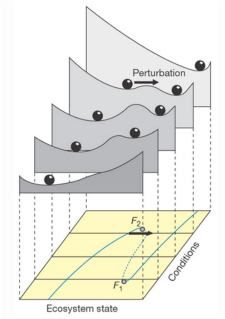
Images: http://resac.gis.umn.edu/water/regional_water_clarity/content/project_summary.htm

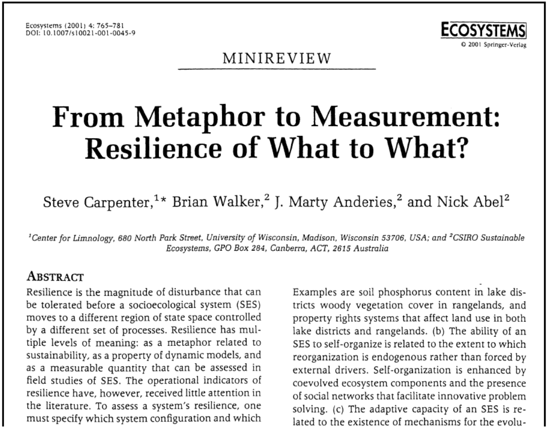
Translating to Dynamical Systems



Scheffer et al. (2001)

Translating to Dynamical Systems





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ECOSYSTEMS
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MINIREVIEW

From Metaphor to Measurement: Resilience of What to What?

Steve Carpenter,[1]* Brian Walker,[2] J. Marty Anderies,[2] and Nick Abel[2]

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ABSTRACT

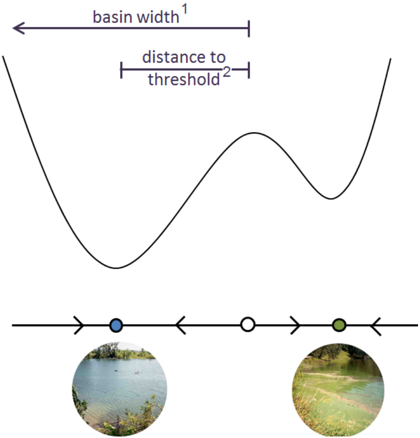
Resilience is the magnitude of disturbance that can be tolerated before a socioecological system (SES) moves to a different region of state space controlled by a different set of processes. Resilience has multiple levels of meaning: as a metaphor related to sustainability, as a property of dynamic models, and as a measurable quantity that can be assessed in field studies of SES. The operational indicators of resilience have, however, received little attention in the literature. To assess a system's resilience, one must specify which system configuration and which

Examples are soil phosphorus content in lake districts woody vegetation cover in rangelands, and property rights systems that affect land use in both lake districts and rangelands. (b) The ability of an SES to self-organize is related to the extent to which reorganization is endogenous rather than forced by external drivers. Self-organization is enhanced by coevolved ecosystem components and the presence of social networks that facilitate innovative problem solving. (c) The adaptive capacity of an SES is related to the existence of mechanisms for the evolu-

Scheffer et al. (2001)

Carpenter et al. (2001)

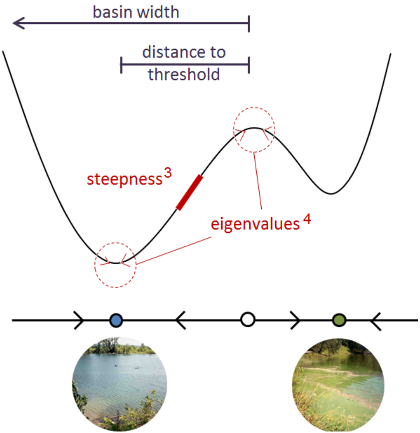
Resilience to state variable perturbations



[1] Levin & Lubchenco (2008), Walker et al. (2004)

[2] Beisner et al. (2003a)

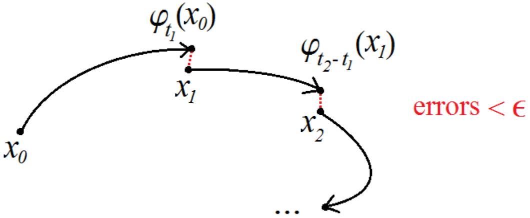
Resilience to state variable perturbations



[3] Beisner et al. (2003b), Levin & Lubchenco (2008)
[4] Ives & Carpenter (2007), Pimm (1984)

ϵ-pseudo-orbits

Classically defined for flows:



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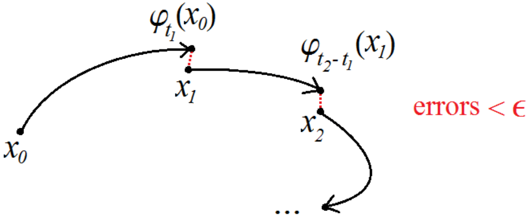


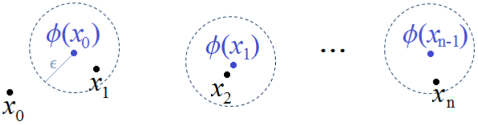
Fix a disturbance period, τ:

$$\varphi(t,x) : \mathbb{R} \times X \to X \qquad \rightsquigarrow \qquad \varphi^{\tau}(x) : X \to X$$

Following McGehee (1988): Let X be a locally compact metric space and $\phi : X \to X$ a continuous map.

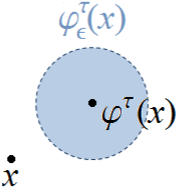
Definition

An ϵ-**pseudo-orbit** of length n is a sequence of points $(x_0, x_1, \ldots, x_n)$ in the state space X satisfying $d(\phi(x_{k-1}), x_k) < \epsilon$, for $k = 1, 2, \ldots n$

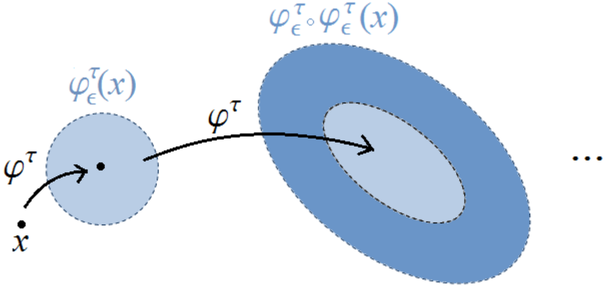


Consider the multi-valued map

$$\varphi_\epsilon^\tau : X \to \mathscr{P}(X)$$
$$x \mapsto \{y \in X : d(y, \varphi^\tau(x)) < \epsilon\}$$



Let $$P_\epsilon(S) = \bigcup_{n=0}^{\infty} (\varphi_\epsilon^\tau)^n (S)$$



Chain intensity of attraction

Definition

The **chain intensity** of an attractor A is

$$\mu(A) \equiv \sup\{\epsilon : P_\epsilon(A) \subset K \subset \mathcal{D}(A), \text{ where } K \text{ is compact}\}$$

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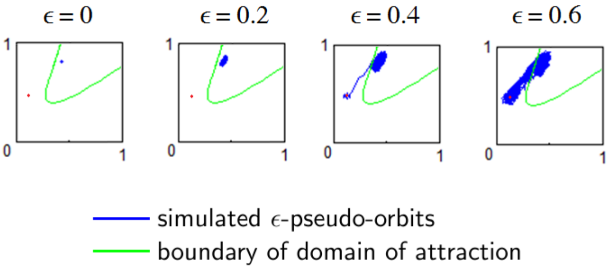
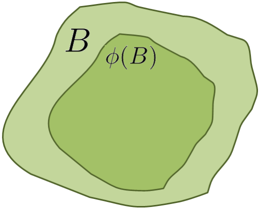
Chain intensity is amenable to numerical approximation:



Attractor blocks

Definition

An **attractor block** for a map ϕ is a nonempty, compact set B such that $\phi(B) \subset \text{int}(B)$.



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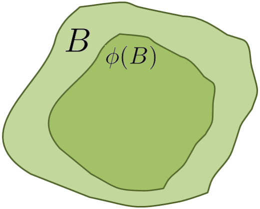


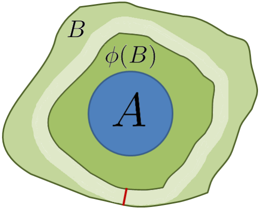
Theorem

For a continuous map on a locally compact metric space,

- *every attractor block contains an attractor in its interior, and*
- *all attractors are the maximal invariant set inside some attractor block.*

Definition

$\beta(B) \equiv \sup\{\epsilon : \varphi_{\epsilon}^{\tau}(B) \subset B\}$



Intensity of attraction

Definition

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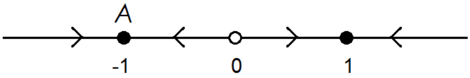
Theorem

$$\mu(A) = \nu(A)$$

Towards a resilience profile

Example

$\dot{x} = x - x^3$



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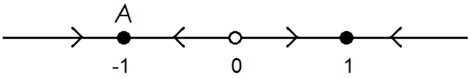

$$\mu^\tau(A) = \nu^\tau(A) = \sup_{x \in (-1,0)} \{x - \varphi^\tau(x)\}$$

Towards a resilience profile

Example

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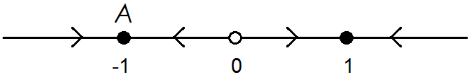

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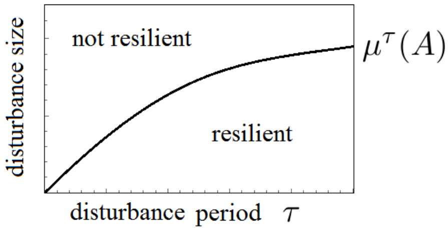
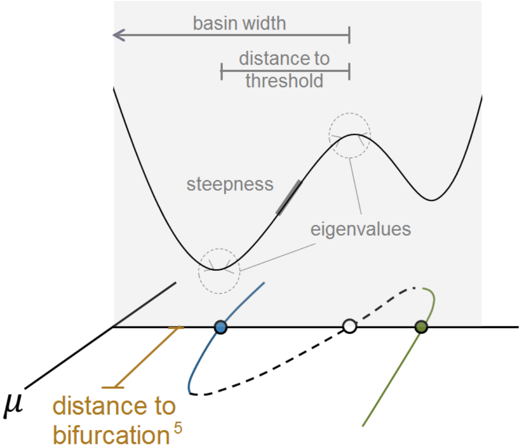
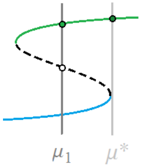


Quantifying resilience to parameter changes



[5] Ives & Carpenter (2007)

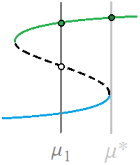
Time-dependent parameter changes



Transient parameter change

Rate-induced tipping

Time-dependent parameter changes



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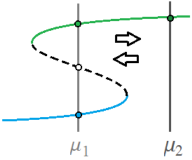

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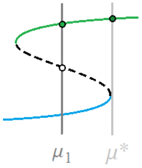

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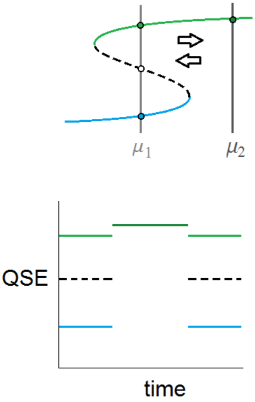

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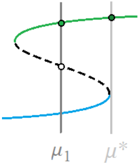

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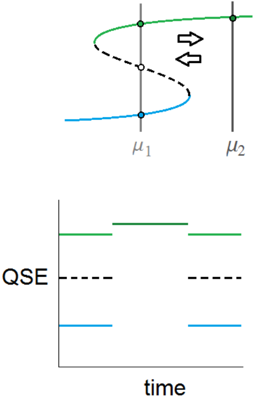

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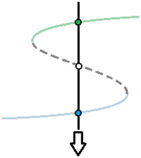

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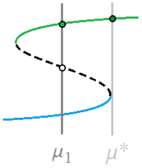

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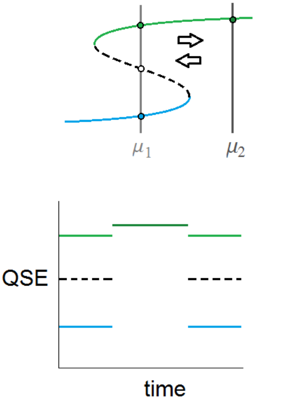

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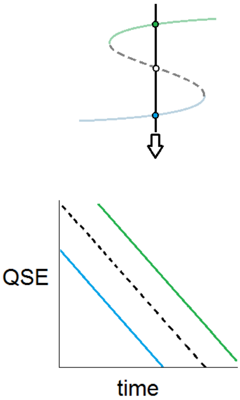
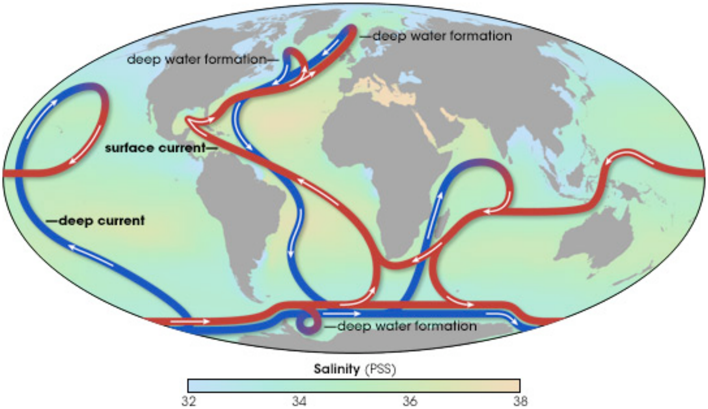


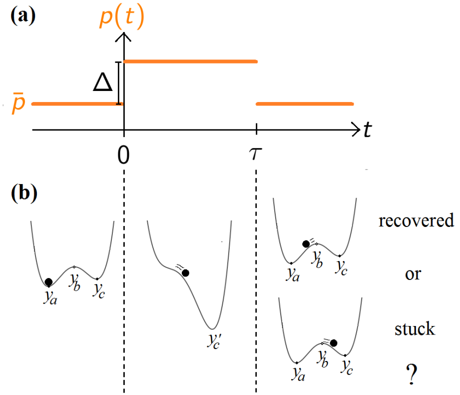
Example: transient change in salinity forcing of ocean circulation



http://earthobservatory.nasa.gov/Features/Paleoclimatology_Evidence/paleoclimatology_evidence_2.php

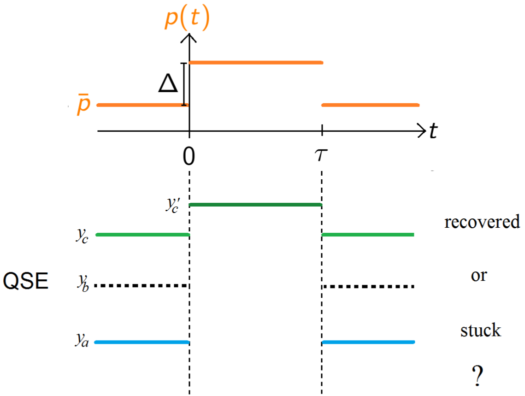
Cessi's 1D model

$$\dot{y} = \bar{p} + \hat{p}(t) - y\left[1 + \mu^2(1-y)^2\right], \text{ where } \hat{p}(t) = \begin{cases} \Delta & 0 \leq t \leq \tau \\ 0 & \text{o'wise} \end{cases}$$

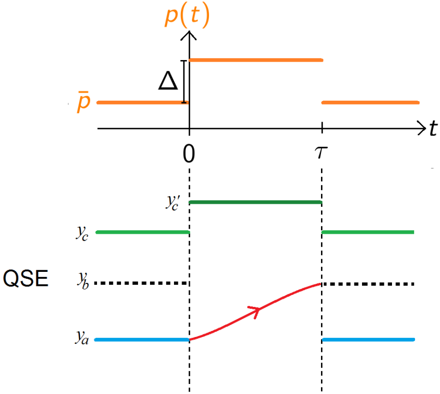


Cessi (1994)

Cessi's 1D model



Calculating critical time in terms of Δ



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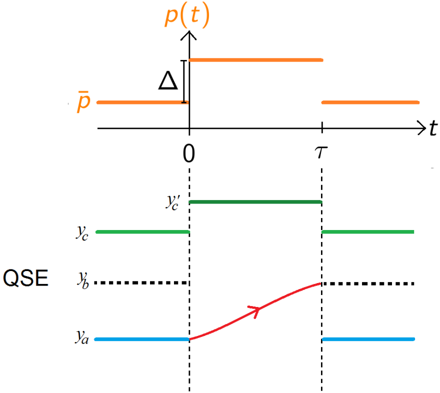

$$\tau_{\text{crit}}(\Delta) = \int_0^{\tau_{\text{crit}}} dt = \int_{y_a}^{y_b} \frac{dy}{\bar{p} + \Delta - y[1 + \mu^2(1-y)^2]}$$

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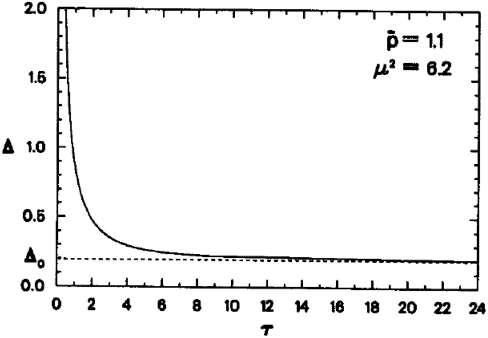
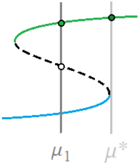
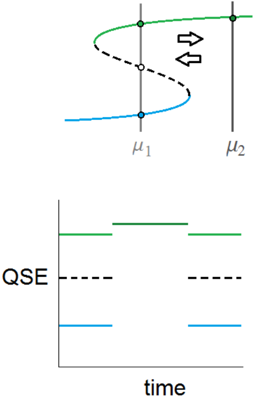


Figure 3 from Cessi (1994)

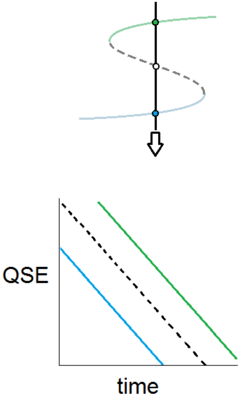
Time-dependent parameter changes



Transient parameter change



Rate-induced tipping

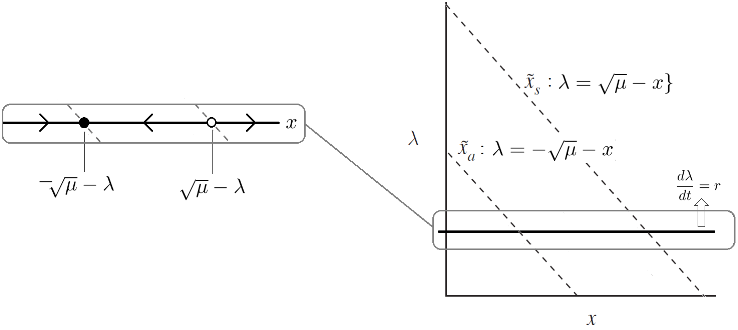


Rate-induced tipping

An example from Ashwin et al. (2012):

$$\frac{dx}{dt} = (x+\lambda)^2 - \mu \qquad (\mu > 0 \text{ fixed})$$

with $$\frac{d\lambda}{dt} = r$$



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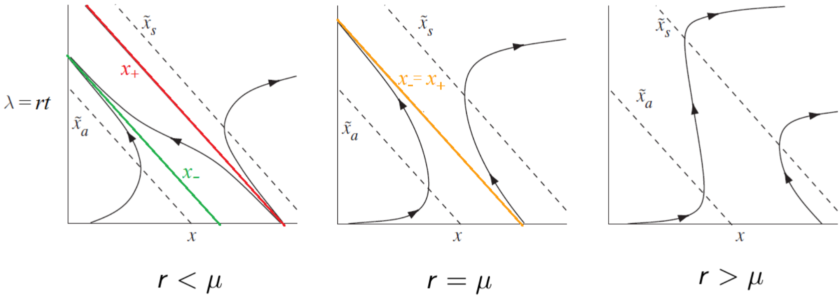
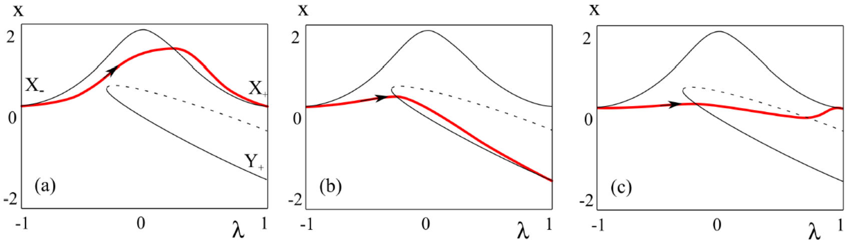


adapted from Figure 3 of Ashwin et al. (2012)

Asymptotically autonomous parameter shifts

An example from Ashwin et al. (2015):

$$\frac{dx}{dt} = -\left((x - 0.25 + b\lambda)^2 - 0.4\tanh(\lambda + 0.3)\right)(x - K/\cosh(3\lambda))$$
$$\frac{d\lambda}{dt} = -r(\lambda + 1)(\lambda - 1)$$



Future directions

- Develop a continuous analogue to intensity of attraction
- Generalize Cessi's method to higher dimensions
- Generalize Cessi's method to smooth parameter changes

Develop a continuous analogue to intensity of attraction

	Maps	**Flows**
Space X	locally compact, metric	locally compact, metric
Disturbed Trajectories	$\phi_\epsilon(x) : X \to \mathscr{P}(X)$	$\varphi_r(t,x) : \mathbb{R} \times X \to \mathscr{P}(X)$ flow on $f+g$, $\lvert g \rvert < r$
Region of Accessibility	$P_\epsilon(A)$	$P_r(A)$
Chain Intensity	$\sup\{\epsilon : P_\epsilon(A) \subset K \subset \mathcal{D}(A)\}$	$\sup\{r : P_r(A) \subset K \subset \mathcal{D}(A)\}$
Attractor Block	compact, nonempty, $\phi(B) \subset \text{int}(B)$	compact, nonempty, $\varphi^t(B) \subset \text{int}(B)\ \forall t > 0$
Intensity	$\sup\{\beta(B)\}$	

Generalize Cessi's method to higher dimensions

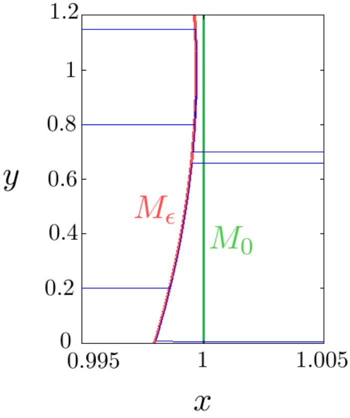
Cessi's 1D system

$$\dot{y} = p - y\left[1 + \mu^2(1-y)^2\right]$$

came from

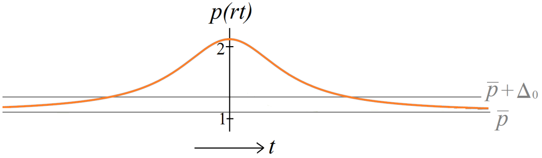
$$x' = -(x-1) - \epsilon x[1 + \mu^2(x-y)^2]$$
$$y' = \epsilon\left(p - y[1 + \mu^2(x-y)^2]\right)$$

via fast-slow reduction.



Generalize Cessi's method to smooth parameter changes

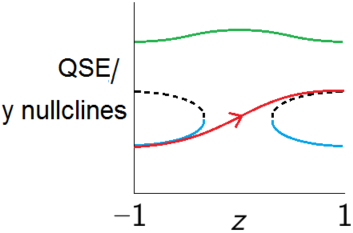
e.g. $p(rt) = \bar{p} + \frac{\Delta}{1+(rt)^2}$

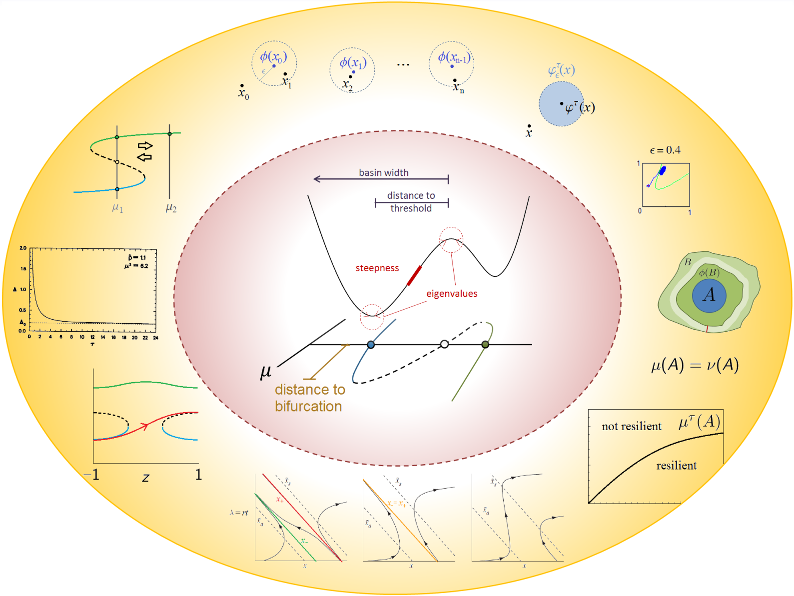


Let $\dot{t} = 1$

$z = \frac{2}{\pi} tan^{-1}(t)$

$\Longrightarrow$





$\phi(x_0)$
x_0
x_1
$\phi(x_1)$
x_2
...
$\phi(x_{n-1})$
x_n
$\varphi_\epsilon^\tau(x)$
$\varphi^\tau(x)$
x
μ_1
μ_2
$\epsilon = 0.4$
basin width
distance to threshold
steepness
eigenvalues
μ
distance to bifurcation
B
$\phi(B)$
A
$\mu(A) = \nu(A)$
-1
z
1
not resilient
$\mu^\tau(A)$
resilient
$\lambda = rt$

References

Ashwin et al. (2015) Parameter shifts for nonautonomous systems in low dimension: bifurcation and rate-induced tipping. *arXiv*, 1506.07734v1

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Thank you.

Questions?