Challenges in Adapting the Budyko Energy Balance Model to Pluto

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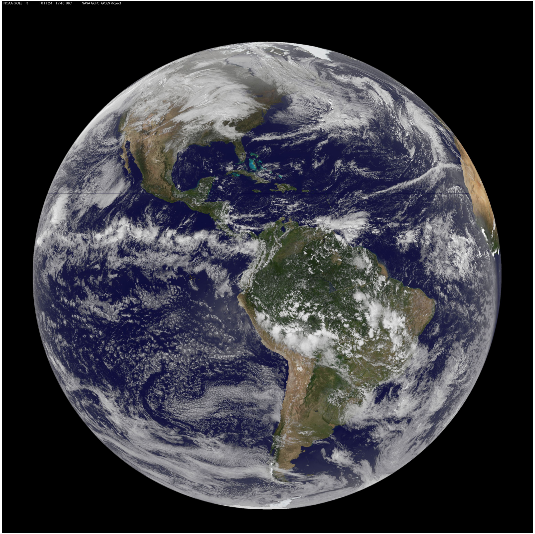
Budyko's Energy Balance Model



$$R\frac{\partial T}{\partial t} = Qs(y)(1 - \alpha(y,\mu)) - (A + BT) + C(\overline{T} - T)$$

Budyko's Energy Balance Model for Pluto?

$$R\frac{\partial T}{\partial t} = \ldots ?$$

Some Particularly Challenging Challenges

- Determining appropriate parameter values
  - Two parameters may be especially difficult to determine
- Understanding “ice line” dynamics
  - What kind of ice are we talking about?
  - How should the ice lines be oriented?

Budyko's Equation

"change in temperature over time"

$$= \text{"energy in"} - \text{"energy out"} + \text{"energy transfer"}$$

$$R\frac{\partial T}{\partial t} = Qs_{\beta}(y)(1 - \alpha(y, \mu)) - (A + BT) + C(\overline{T} - T)$$

- Change in temperature by sine of latitude (y)
- R is the Earth's heat capacity

Budyko's Equation

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= "energy in" − "energy out" + "energy transfer"

$$R\frac{\partial T}{\partial t} = Qs_\beta(y)(1 - \alpha(y, \mu)) - (A + BT) + C(\overline{T} - T)$$

- Incoming solar radiation
- Q is the integrated total solar input into the system
- Distribution of insolation (s_β) depends on sine of latitude
- Albedo (α) dependent on both sine of latitude and the location of the ice lines ($\mu = (\mu_N, \mu_S)$)

Budyko's Equation

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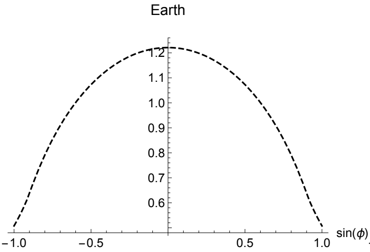

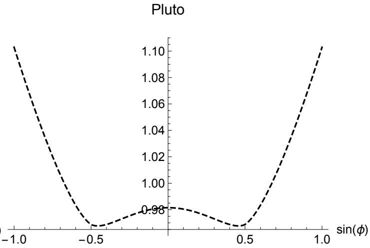

Budyko's Equation

"change in temperature over time"

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$$R\frac{\partial T}{\partial t} = Qs_\beta(y)(1-\alpha(y,\mu)) - (A+BT) + C(\overline{T} - T)$$

- Outgoing radiation
- Linear approximation of Stefan-Boltzmann black body radiation

Budyko's Equation

"change in temperature over time"
= "energy in" − "energy out" + "energy transfer"

$$R\frac{\partial T}{\partial t} = Qs_\beta(y)(1 - \alpha(y,\mu)) - (A + BT) + C(\overline{T} - T)$$

- Zonal energy transfer
- $C = 1.6\,B$
- $\overline{T}$ is global average temperature, dependent on the ice lines μ

Determining Parameter Values

$$R\frac{\partial T}{\partial t} = Qs_\beta(y)(1-\alpha(y,\mu)) - (A+BT) + C(\overline{T}-T)$$

- In the Earth model, A and B are determined empirically from satellite data
- We will use the Stefan-Boltzman law:

$$I(y) = \delta\sigma T^4 \approx \delta\sigma T_0^4\left(1+\frac{4(T-T_0)}{T_0}\right)$$

$$A = \delta\sigma T_0^4, \qquad B = \frac{4A}{T_0}$$

Determining Parameter Values

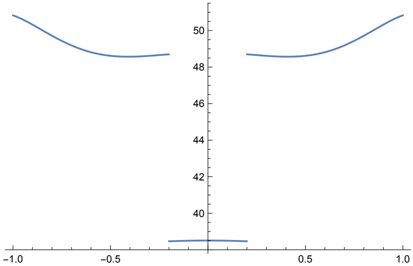
$$A = \delta \sigma T_0^4, \qquad B = \frac{4A}{T_0}$$

- $0 \leq \delta \leq 1$ is determined by composition of surface and atmosphere
- Pluto has an atmosphere ... right now

- δ may change drastically over the course of a Pluto year [Stansberry and Yelle]

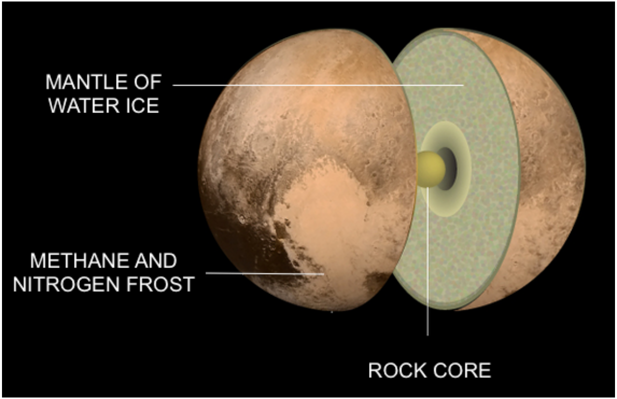
Determining Parameter Values

- Option #1:
  - Take $\delta = .5$
  - Take $T_0 = 36\,^\circ$K so that the temperature profile is in an appropriate range



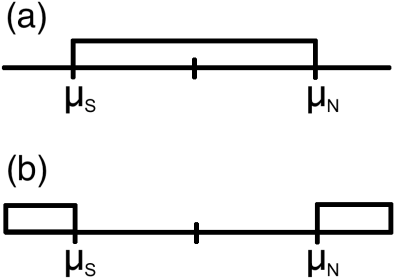
- Option #2:
  - Take $T_0 = 37.5\,^\circ$K, Pluto's black body temperature
  - Take $\delta = .45$ so that the temperature profile is in an appropriate range

Ice Line Dynamics: Pluto's Ice



- Nitrogen ice is the most abundant ice on Pluto's surface by a factor of 50 [Owen]
- Darker regions on Pluto's surface are composed of tholins [Betz]

Ice Line Dynamics: The Albedo Function



$$\alpha(y,\mu) = \begin{cases} \alpha_1 & y < \mu_S \\ \alpha_2 & \mu_S < y < \mu_N \\ \alpha_1 & y > \mu_N \end{cases}$$

- Nitrogen ice has albedo of .8 [Stransberry and Yelle]
- Tholins have albedos ranging from .01 to .15 [Cruikshank]

Ice Line Dynamics: Critical Temperature

- At Earth's surface pressure, 1 atm, nitrogen freezes at 63.15 $^\circ$K
- Pluto's surface pressure is 10^{-6} atm and average temperature is 44 $^\circ$K
- Nitrogren's phase properties are not well understood at such low temperatures and pressures
- Satorre et al. give a freezing temperature of 22 $^\circ$K at 10^{-10} atm

Equilibrium Solutions

Equilibrium solutions, T^*, will satisfy

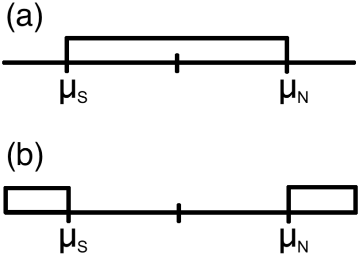
$$0 = Qs_\beta(y)(1-\alpha(y,\mu)) - (A + BT^*(y,\mu)) + C(\overline{T^*}(\mu) - T^*(y,\mu)).$$

To find T^*, first solve for $\overline{T}^*$ by integrating in y.

The resulting equilibrium temperature profile is

$$T^*(y,\mu) = \frac{1}{B+C}(Qs_\beta(y)(1-\alpha(y,\mu)) - A + C\bar{T}^*(\mu))$$

Equilibrium Solutions



$$\alpha(y,\mu) = \begin{cases} \alpha_1 & y < \mu_S \\ \alpha_2 & \mu_S < y < \mu_N \\ \alpha_1 & y > \mu_N \end{cases}$$

The lefthand and righthand limits at the southern ice line are

$$T^*(\mu_{S-},\mu) = \frac{1}{B+C}(Qs_\beta(\mu_S)(1-\alpha_1) - A + C\bar{T}^*(\mu))$$

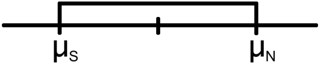
$$T^*(\mu_{S+},\mu) = \frac{1}{B+C}(Qs_\beta(\mu_S)(1-\alpha_2) - A + C\bar{T}^*(\mu))$$

At equilibrium

$$T_c = \frac{T^*(\mu_{S-},\mu) + T^*(\mu_{S+},\mu)}{2} = T^*(\mu_S,\mu)$$

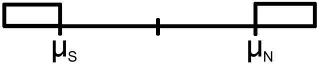
Dynamics of the Ice Line

(a)



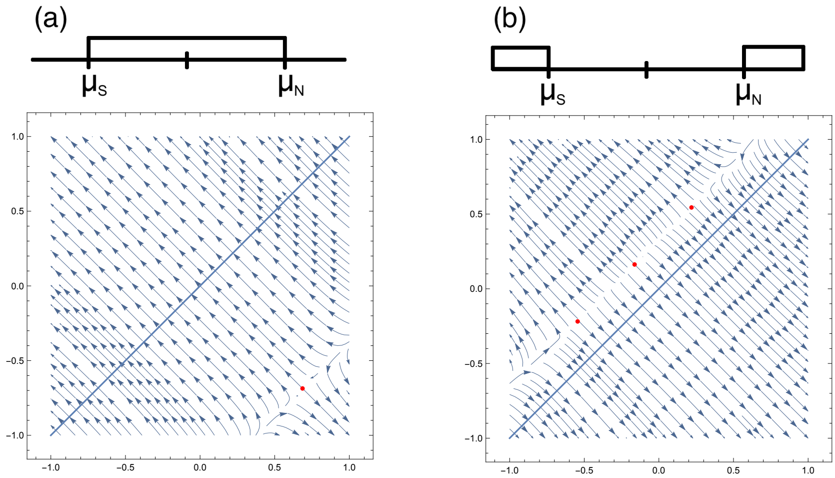
$$\begin{bmatrix} \dot{\mu}_S \\ \dot{\mu}_N \end{bmatrix} = \begin{bmatrix} T^*(\mu_S, \mu) - T_c \\ T_c - T^*(\mu_N, \mu) \end{bmatrix}$$

(b)

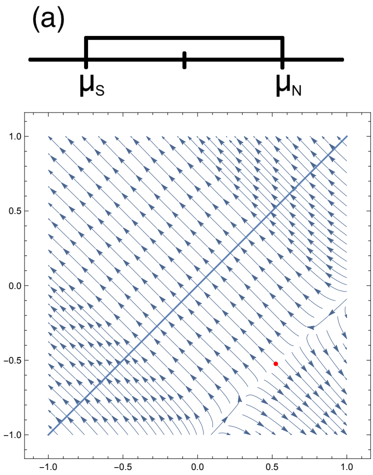


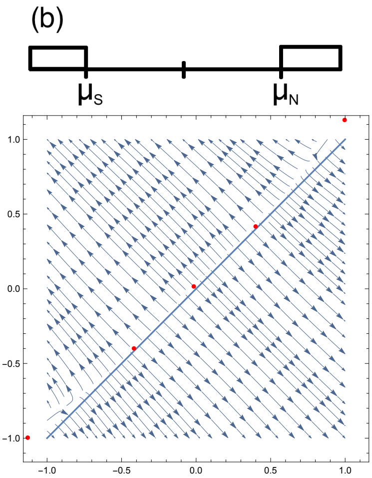
$$\begin{bmatrix} \dot{\mu}_S \\ \dot{\mu}_N \end{bmatrix} = \begin{bmatrix} T_c - T^*(\mu_S, \mu) \\ T^*(\mu_N, \mu) - T_c \end{bmatrix}$$

Vector Field with Equilibria, $T_c = 25\,^\circ\mathrm{K}$

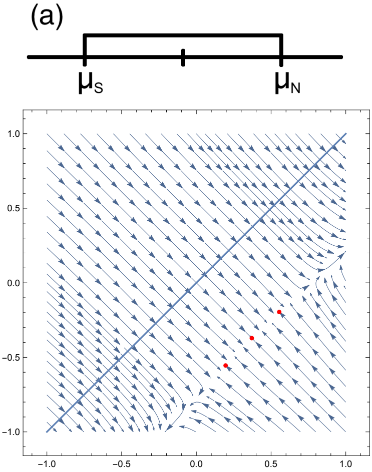


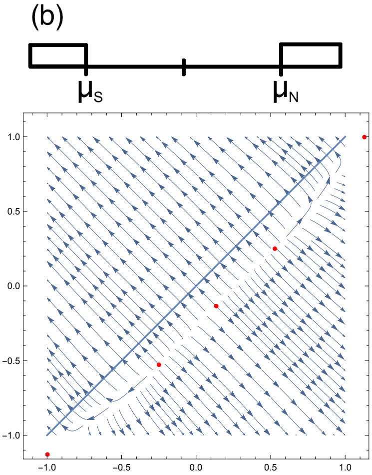
Vector Field with Equilibria, $T_c = 30\,^\circ\mathrm{K}$



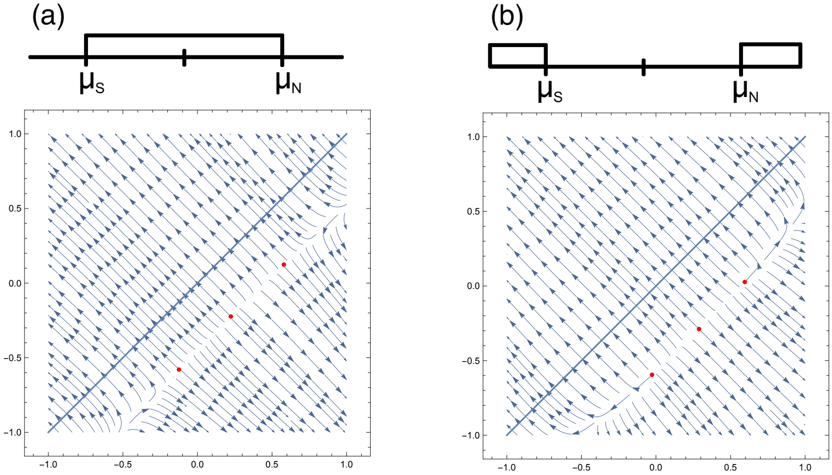


Vector Field with Equilibria, $T_c = 35\,^\circ\mathrm{K}$

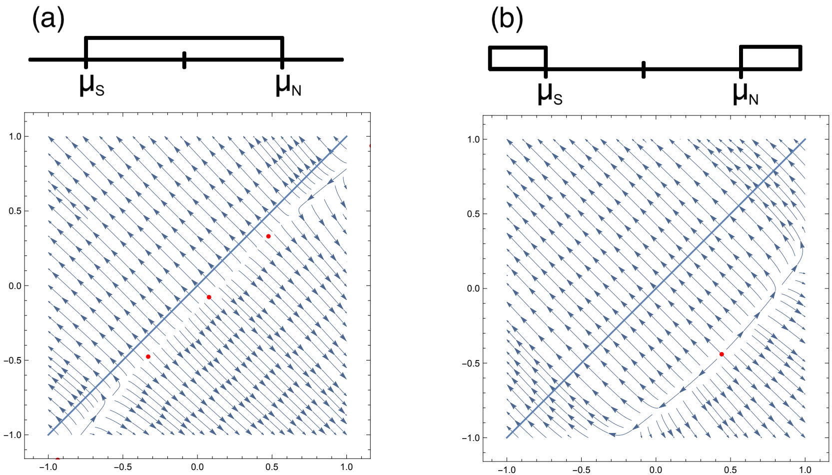




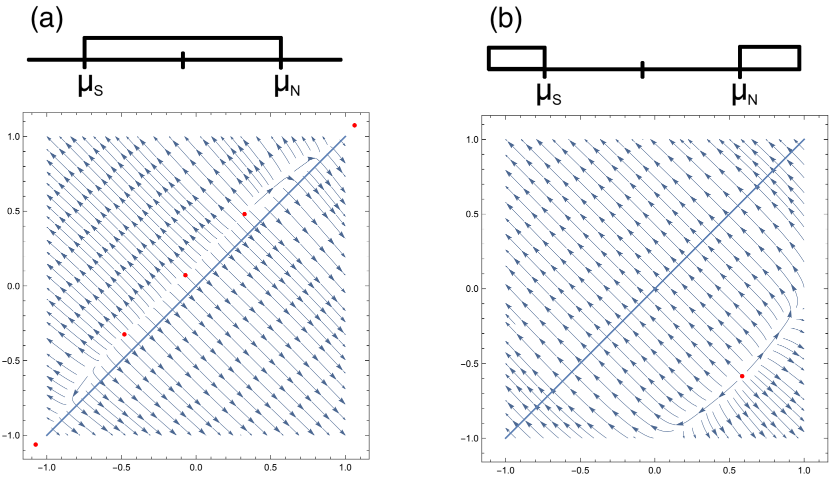
Vector Field with Equilibria, $T_c = 40\,^\circ\mathrm{K}$



Vector Field with Equilibria, $T_c = 45\,^\circ\mathrm{K}$



Vector Field with Equilibria, $T_c = 50\,^\circ\mathrm{K}$



Take Aways

- There is a lot of uncertainty in some parameter values
  - but we will know more when New Horizons sends its information back
- Pluto's simplified "ice lines" are unstable
  - but we haven't accounted for the fact that Pluto may loose it's atmosphere in part of it's orbit

Looking Forward

- Define a function space and study the dynamics
  - McGehee and Widiasih used a four dimensional function space, but they used a quadratic approximation for insolation
  - We need at least a sixth order Legendre polynomial to capture the proper insolation distribution for Pluto
- Allow for four ice lines
  - Preliminary work with the Budyko model on Earth with four ice lines yield interesting results
  - Could we get stable solutions on Pluto with this configuration?

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