The Mid-Pleistocene Transition in the Glacial Flip-Flop Model

5/22/2019

SIAM DS19

Somyi Baek
Malte Stuecker
Esther Widiasih

Mathematics and Climate Research Network

Image courtesy of Wikimedia Commons

Collaborators

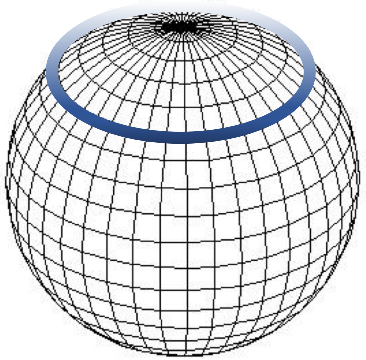
Esther Widiasih
University of Hawaii
West Oahu

Malte Stuecker
Busan National University
South Korea

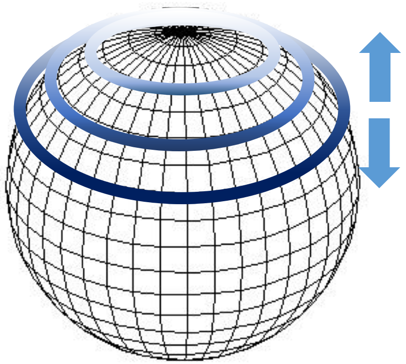
“The Mid Pleistocene Transition from Budyko's Energy Balance Model” (submitted for publication Oct. 2018)

Timeline

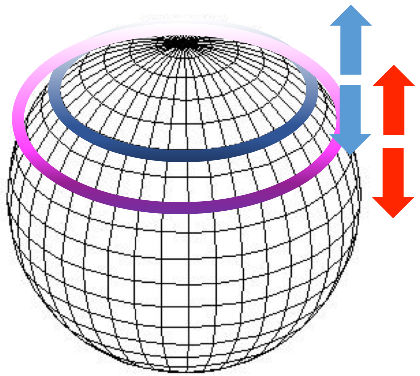
1969	2014	2016	2019
Budyko's EBM	Budyko-Widiasih	Flip-Flop	Flip-Flop (MPT?)



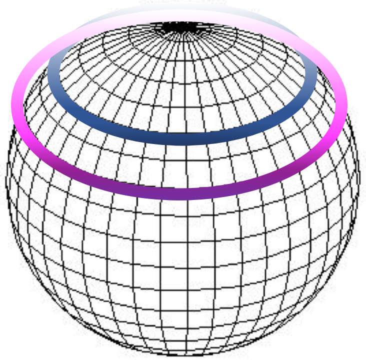
Fixed Ice Line



Dynamic Ice Line

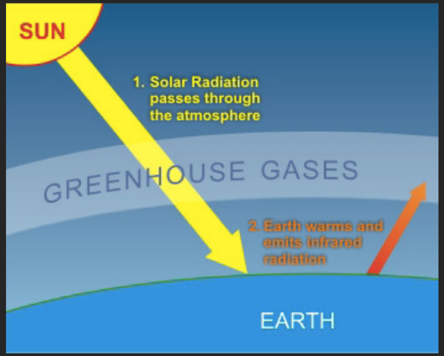


Dynamic Ice Line & Glacier's Edge



Ramped Critical Temperature for MPT

PART 1:
BUDYKO-WIDIASIH MODEL



Budyko-Widiasih Model

$$\frac{\partial}{\partial t}T(t,y) = \frac{1}{R}\left(\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{Absorbed insolation}} - \underbrace{(A+BT(y))}_{\text{OLR}} - \underbrace{C\left(T(y)-\overline{T}\right)}_{\text{transport}}\right)$$

Temperature Equation

$$\frac{d\eta}{dt} = \varepsilon\left[T(\eta) - T_c\right]$$

Ice Line Equation

E. Widiasih, R. McGehee. A Quadratic Approximation to Budyko's Ice-Albedo Feedback Model with Ice Line Dynamics, *SIAM J. Appl. Dyn. Syst.*, March 2014.

Budyko-Widiasih Model: Temperature Equation

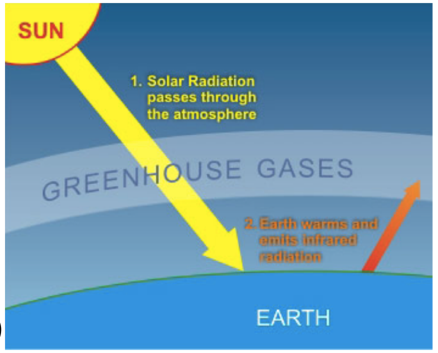
Annual average temp at latitude $y = \sin\,\theta$

$$\frac{\partial}{\partial t} T(t,y) = \frac{1}{R}\left(\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{Absorbed insolation}} - \underbrace{(A+BT(y))}_{\text{Outgoing Longwave Radiation}} - \underbrace{C\,(T(y)-\overline{T})}_{\text{transport}} \right)$$

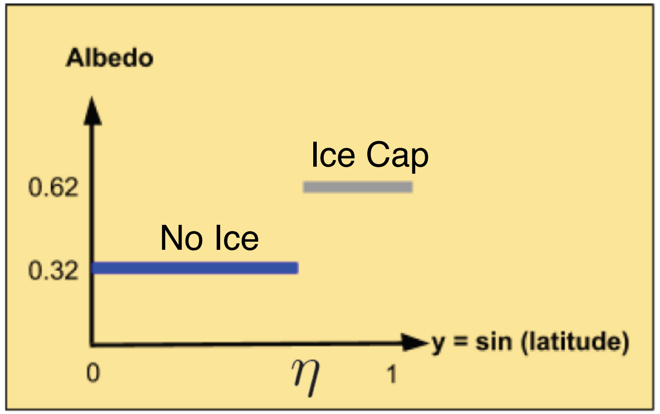
$$\frac{d\eta}{dt} = \varepsilon\,[T(\eta) - T_c]$$

Based on the simple idea:

Temperature Change ~ Energy IN – Energy OUT



(Courtesy of skepticalsciences.com)

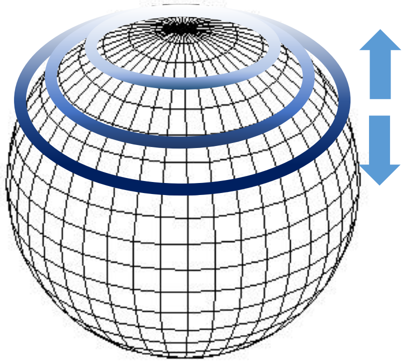


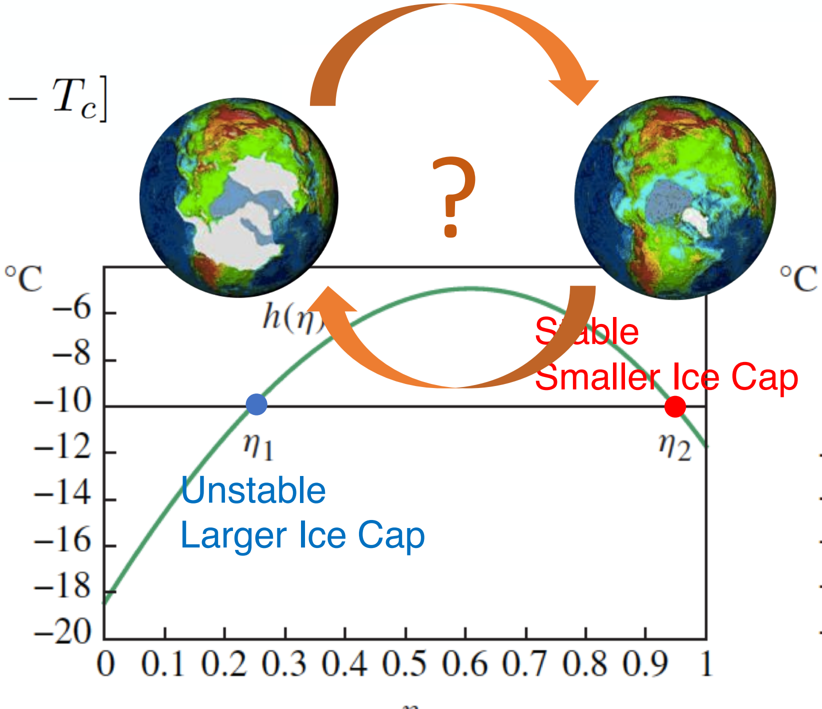
(Courtesy of E. Widiasih)

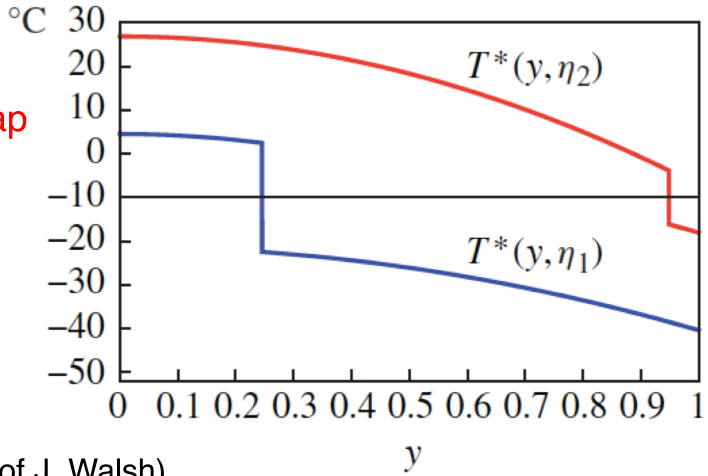
Budyko-Widiasih Model: Ice Line Dynamics

$$\frac{\partial}{\partial t}T(t,y) = \frac{1}{R}\Bigg(\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{Absorbed insolation}} - \underbrace{(A+BT(y))}_{\text{OLR}} - \underbrace{C\left(T(y)-\overline{T}\right)}_{\text{transport}}\Bigg)$$

$$\frac{d\eta}{dt} = \varepsilon\,[T(\eta) - T_c]$$





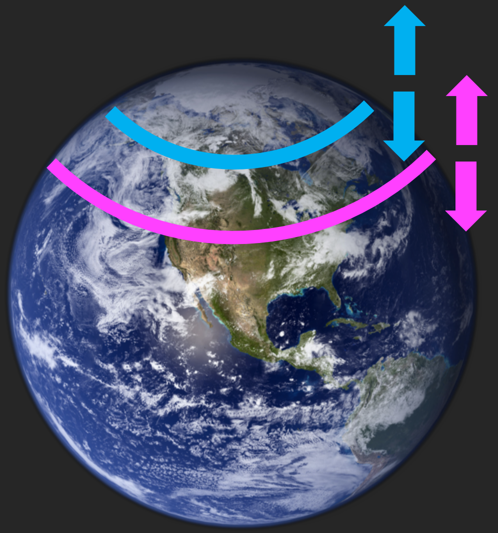


$T(\eta) > T_c$: Ice line moves poleward
$T(\eta) < T_c$: Ice line descends

(Courtesy of J. Walsh)

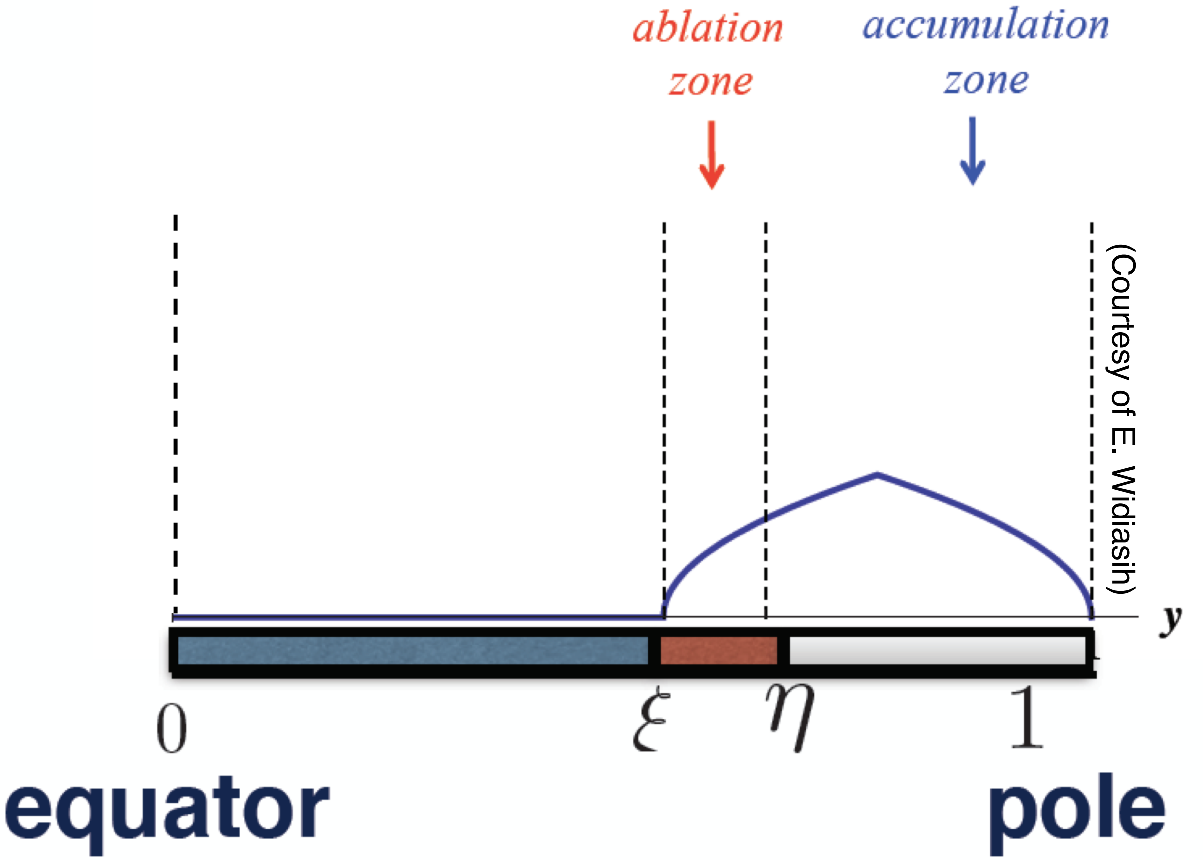
PART 2:

GLACIAL FLIP-FLOP MODEL



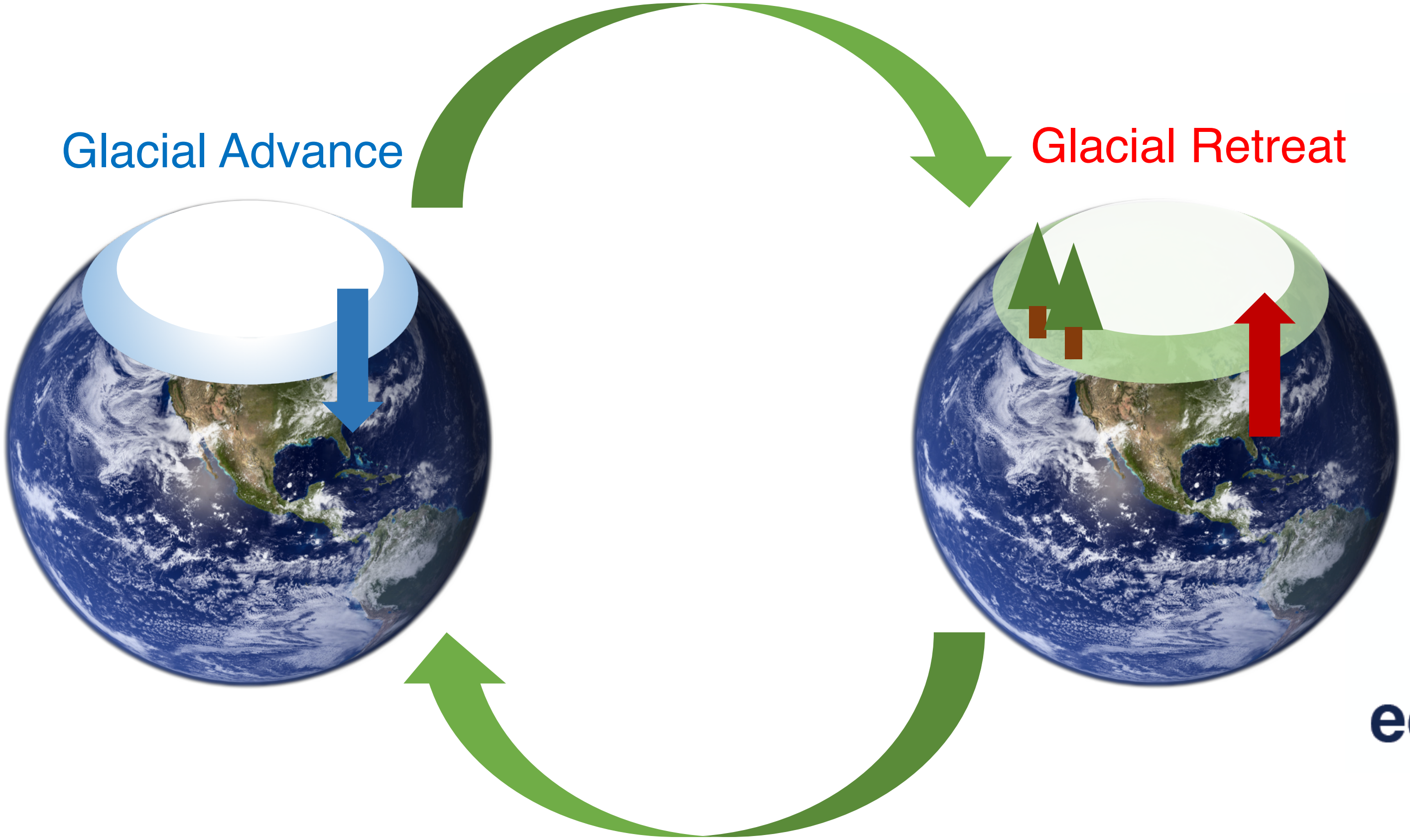
From EBM to Flip-Flop Model: Variables

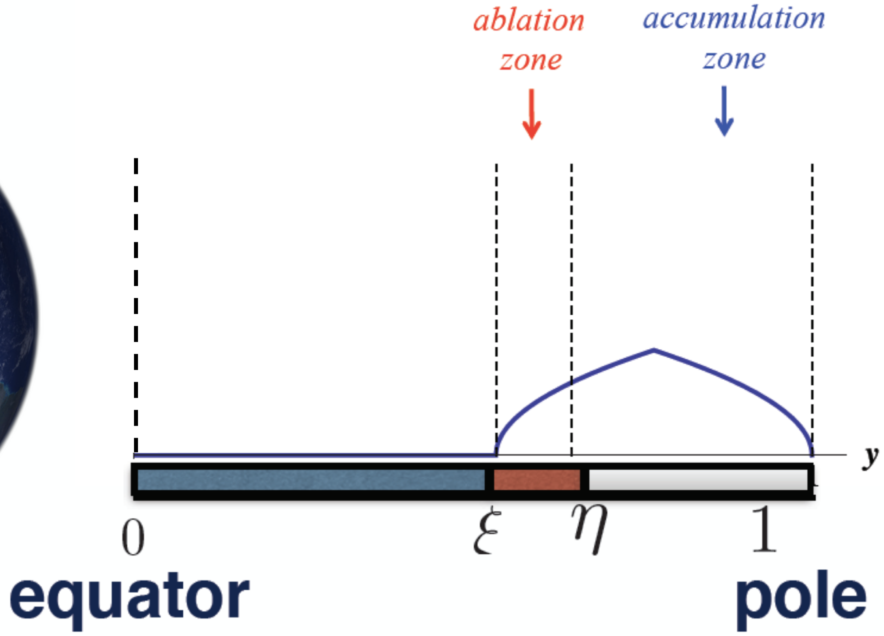
Widiasih's Snow Line η	Snow/ice cover remains throughout the year
(NEW) Glacier's Edge Variable ξ	Snow accumulating to become glacier, or thinner retreating ice



J. Walsh, E. Widiasih, J. Hahn, R. McGehee. Periodic orbits for a discontinuous vector field arising from a conceptual model of glacial cycles. *Nonlinearity* 29.6 (2016): 1843.

Flip-Flop Model's States: Motivation

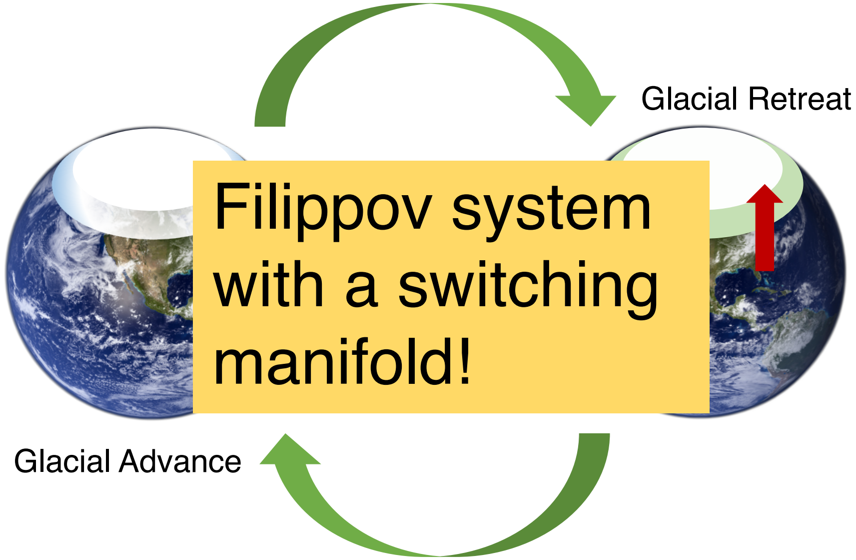




Flip-Flop Model's States: Glacial Advance & Retreat

Glacial Advance

- Global conditions favor ice sheet decay
- Ablation < accumulation
- $T_c = -10°C$



Glacial Retreat

- Global conditions favor ice sheet decay
- Ablation > accumulation
- $T_c = -5.5°C$ (can vary)

$$\frac{\partial}{\partial t}T(t,y) = \frac{1}{R}\left(\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{Absorbed insolation}} - \underbrace{(A+BT(y))}_{\text{OLR}} - \underbrace{C\left(T(y)-\overline{T}\right)}_{\text{transport}}\right)$$

$$\frac{d\eta}{dt} = \varepsilon\left[T(\eta) - T_c\right]$$

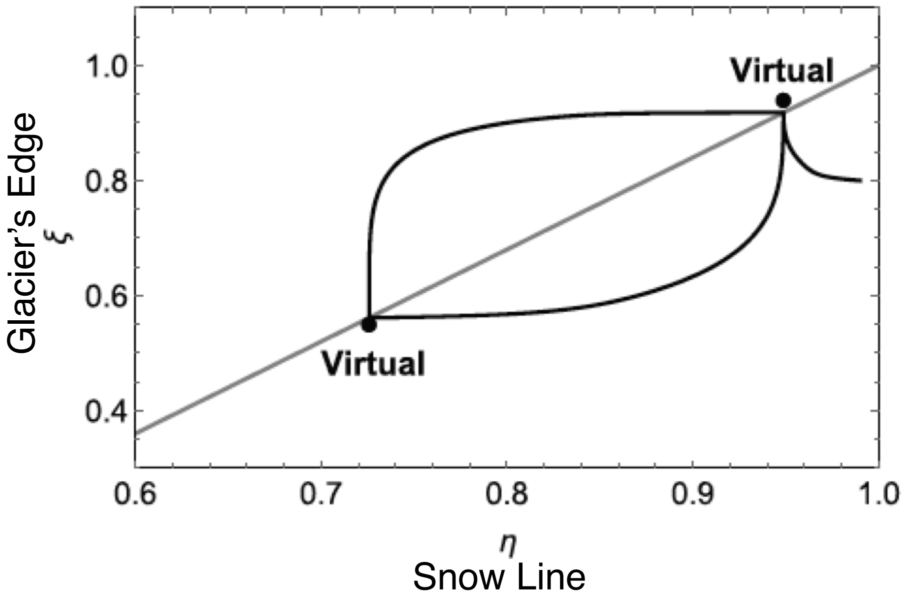
$$\frac{d\xi}{dt} = \epsilon\left[b(\eta-\xi) - a(1-\eta)\right]$$

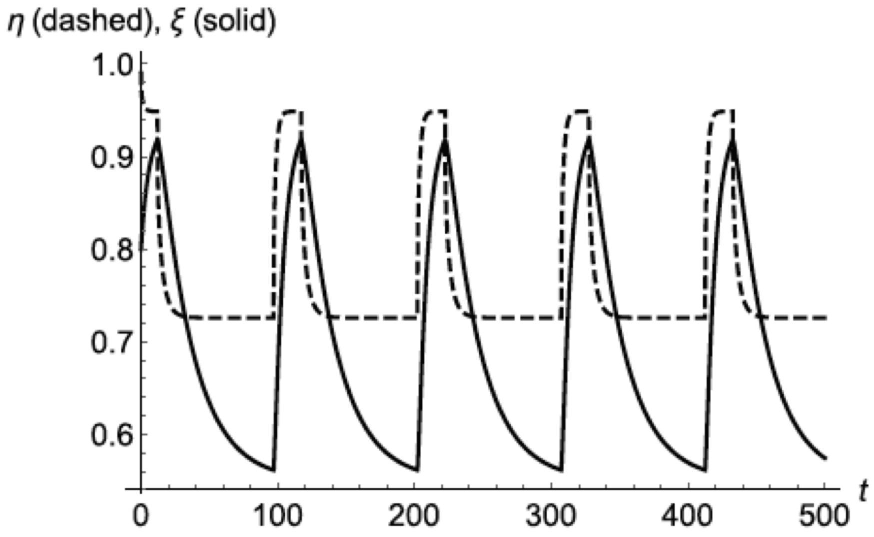
Switching between states at Ablation rate = Accumulation rate

Flip-Flop Model's Dynamics

Theorem(WWHM-2016):

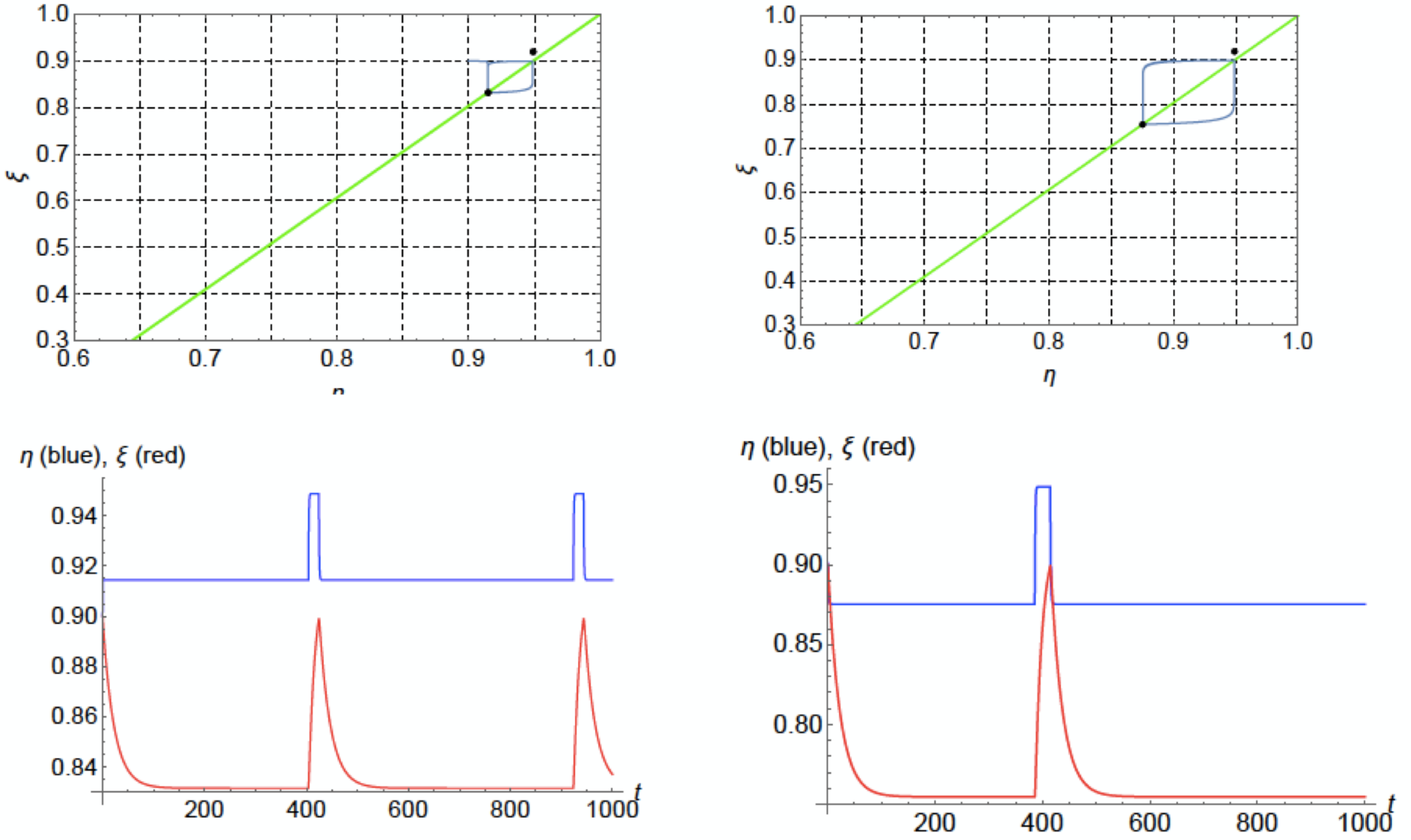
With the standard set of parameters, the Filippov system of the Flip-Flop model admits a unique, attracting periodic orbit





Creating Desired Cycles from Flip-Flop

(Courtesy of E. Widiasih)

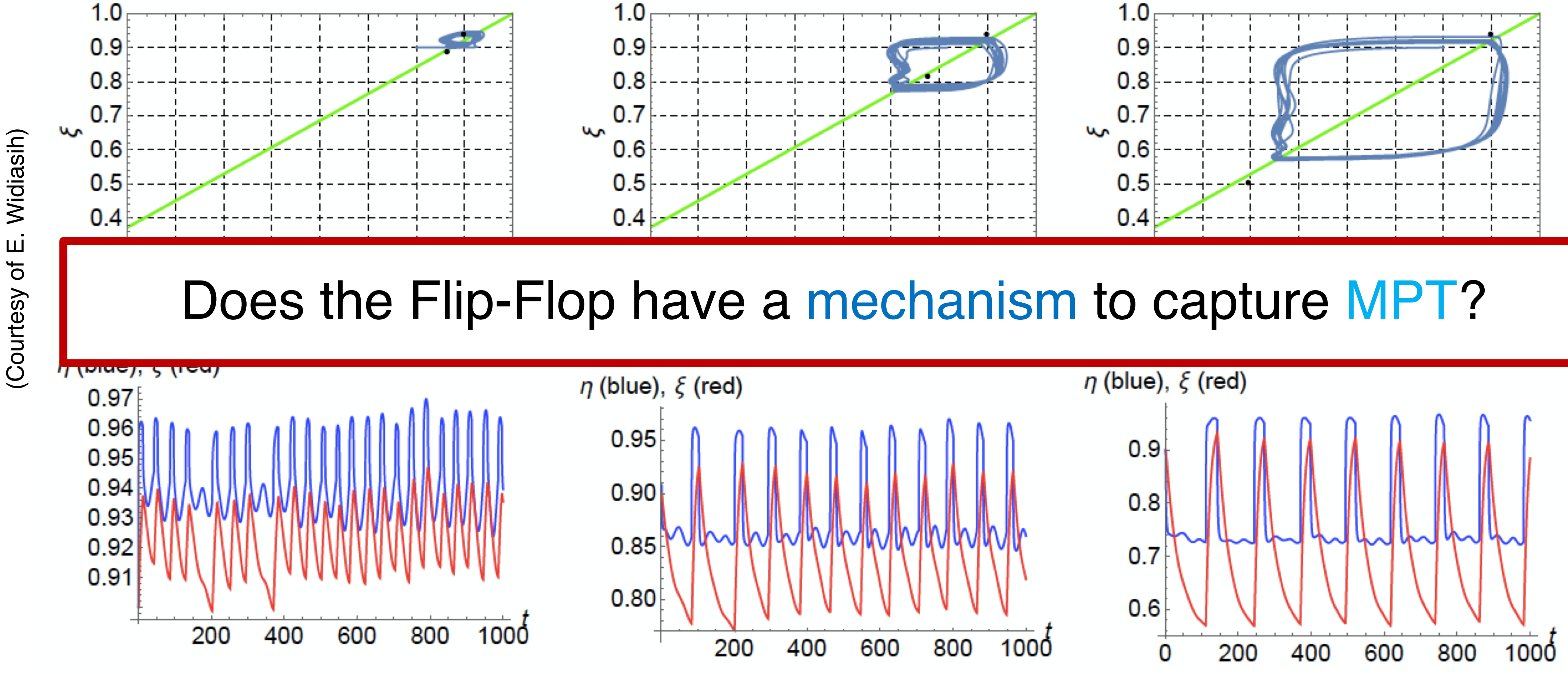


Blue: Ice Line
Red: Glacier's Edge

T_{c_r} = T_c for glacial ret.
T_{c_a} = T_c for glacial adv.

Conjecture: $|T_{c_r} - T_{c_a}|$ drives the orbit's period and amplitude.

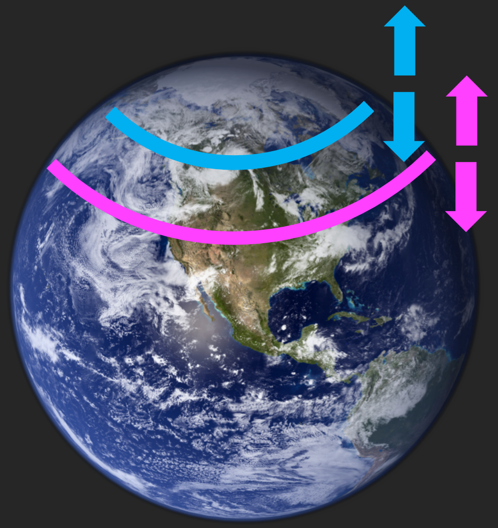
Adding Milankovitch Forcing



Does the Flip-Flop have a mechanism to capture MPT?

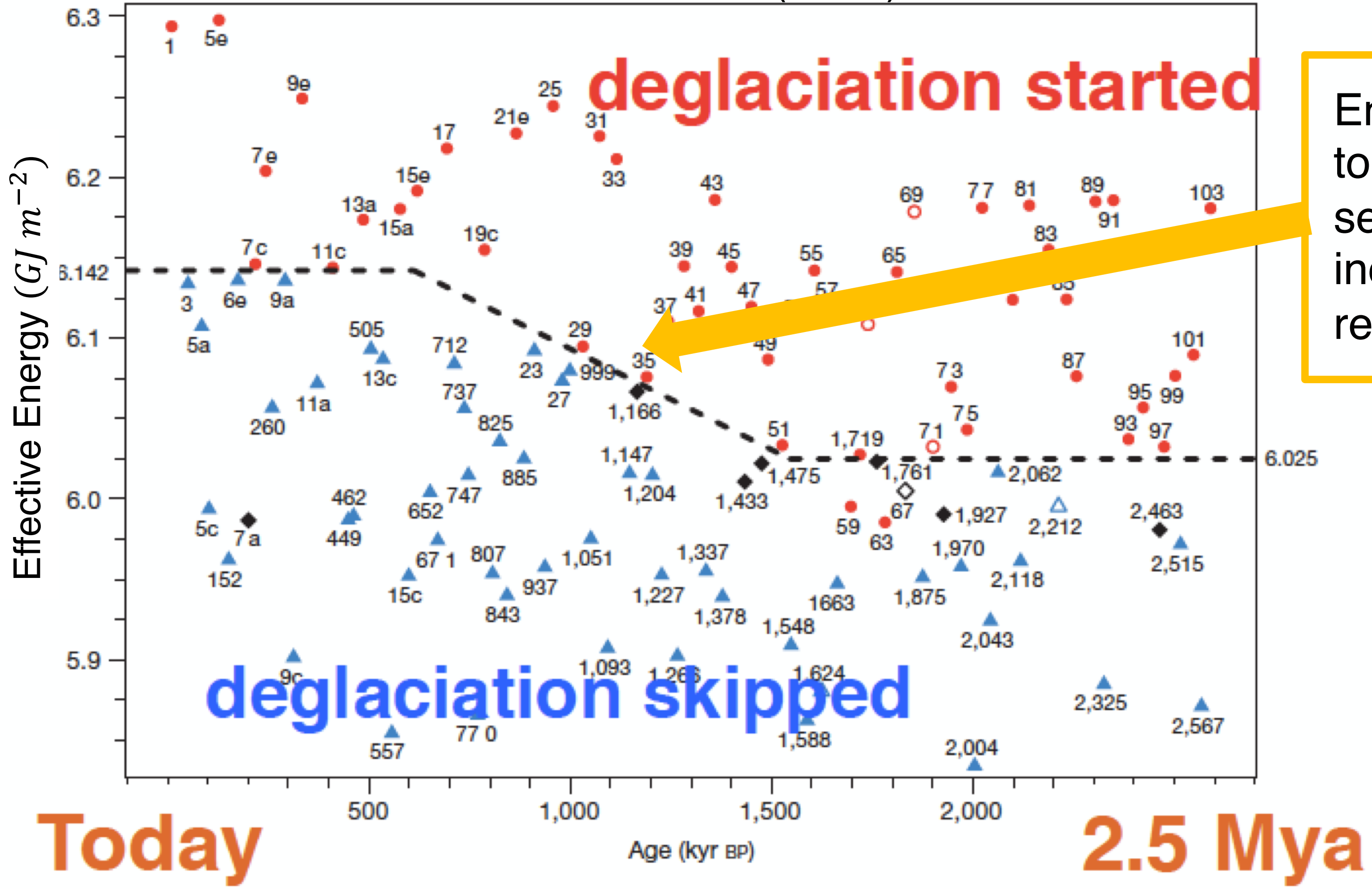
T_{c_r} = -9, T_{c_a} = -10 T_{c_r} = -8, T_{c_a} = -10 T_{c_r} = -7, T_{c_a} = -10

PART 3:
GLACIAL FLIP-FLOP MODEL:
MPT edition



Motivation to tweak Flip-Flop's structure for MPT

Tzedakis et al. (2017)

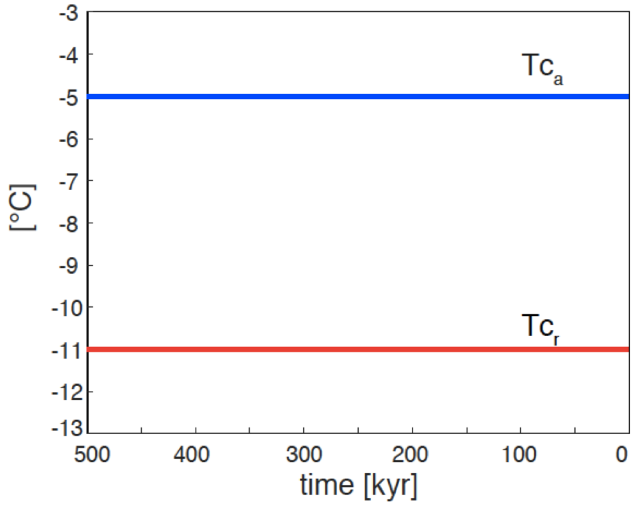


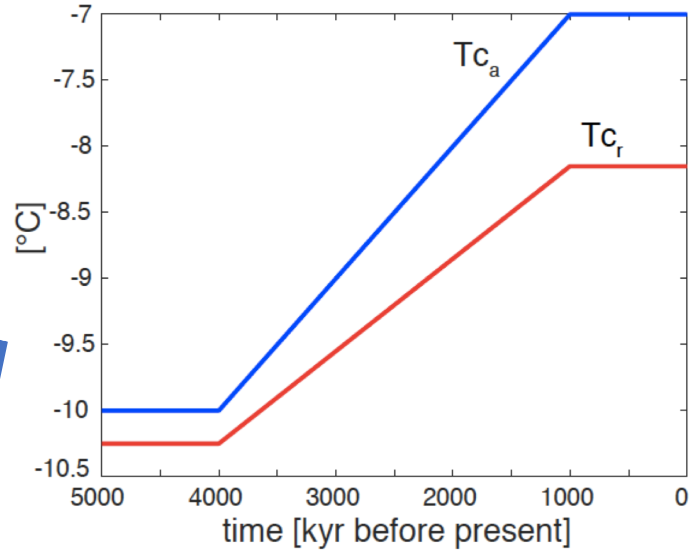
All points plotted are 'caloric summer half-year insolation peaks' at 65° North

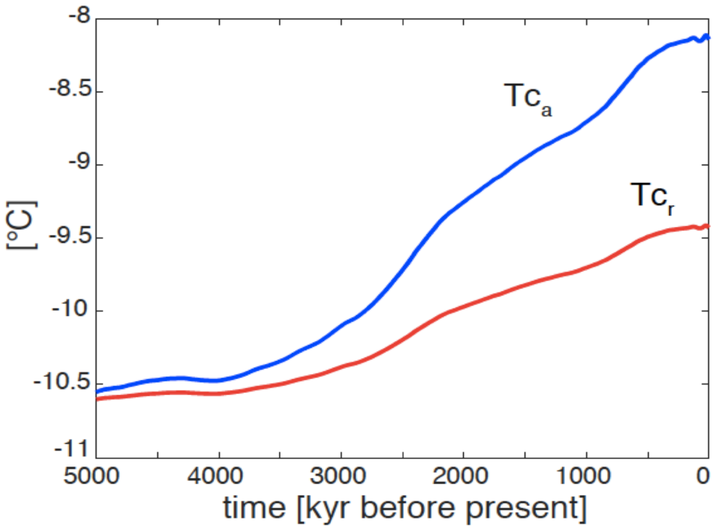
Energy required to deglaciate seem to have increased in the recent years

Motivation to tweak Flip-Flop’s structure for MPT

- Incorporate the idea of “Increased energy required for deglaciation” into our critical temperatures for both



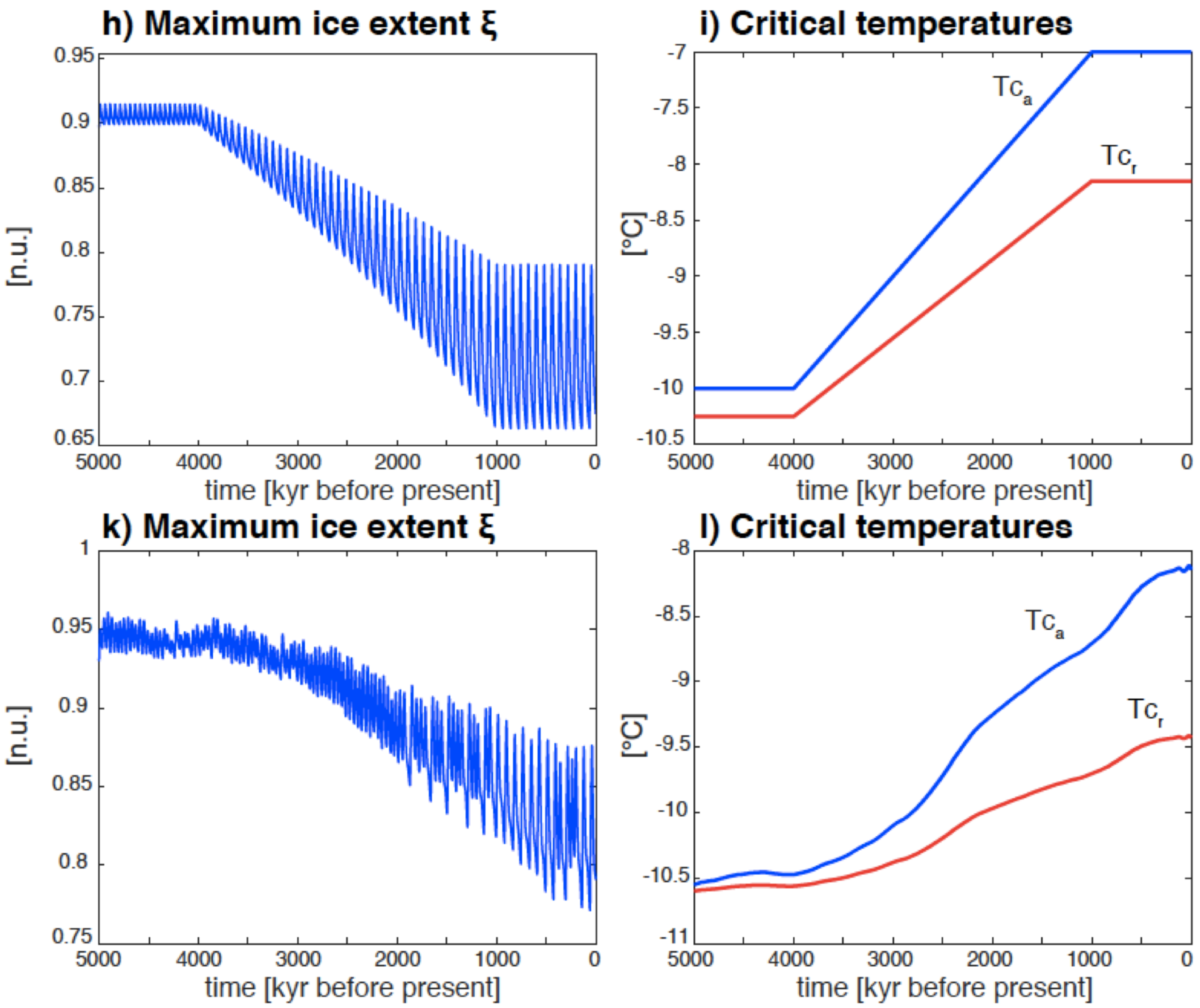




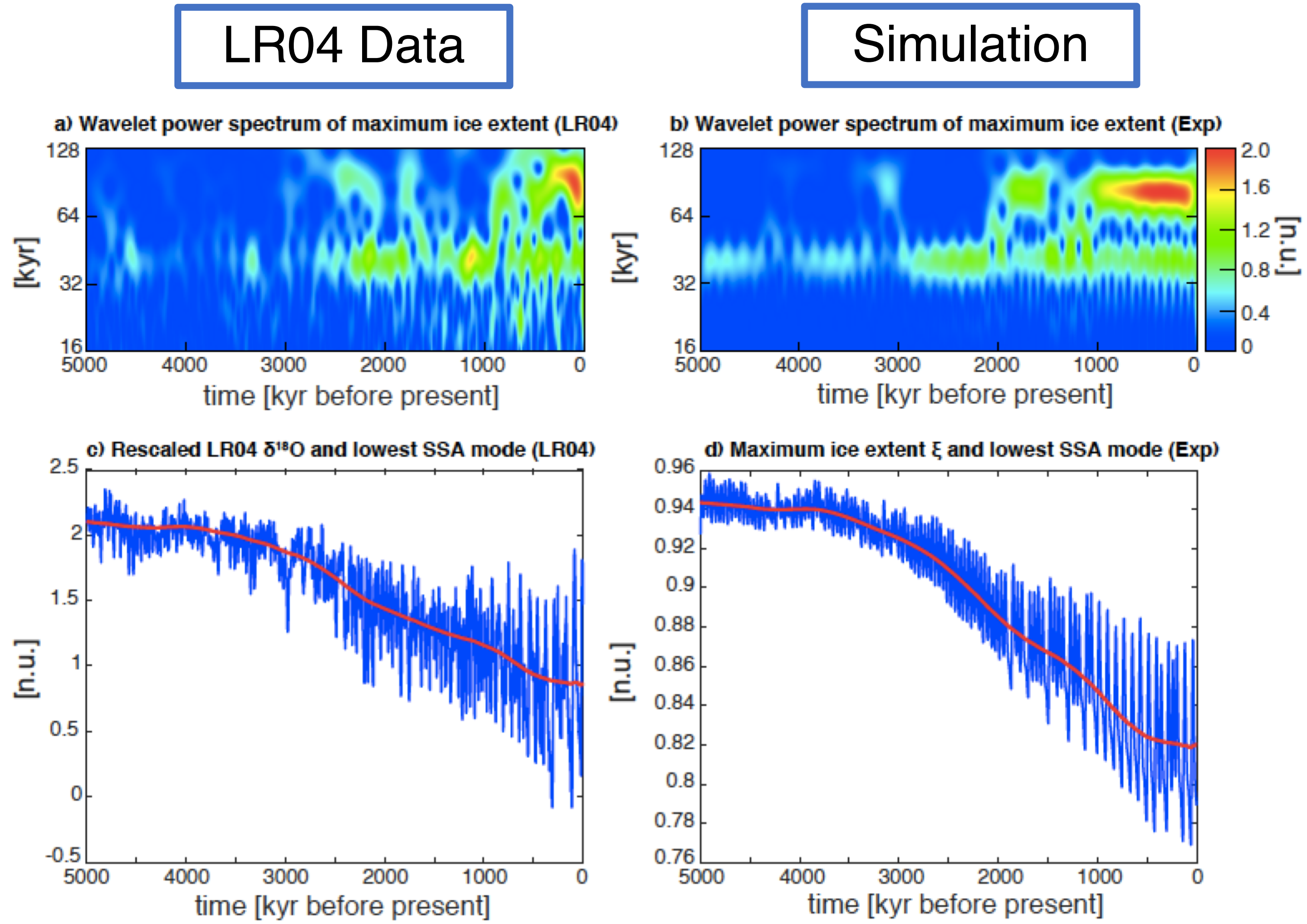
Flip-Flop Model Simulation with modified T_c

Simulation 1

Simulation 2



Checking for MPT in Simulation



Conclusion

- A simple glacial cycle model based on Budyko’s energy balance model is used to simulate the glacial cycles over the past 5 million years
- When a critical parameter of the model is linked to a stack of benthic 18O, the model simulates a realistic Mid Pleistocene Transition
- The critical parameter is the ice forming critical temperature, capturing a connection between the ice-albedo feedback and the temperature-accumulation ablation feedback.

References

- M.I. Budyko, The effect of solar radiation on the climate of the earth, Tellus, 21 (1969), pp. 611–619
- Hans G. Kaper and Hans Engler, Mathematics and Climate, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, Pennsylvania, 2013. (in preparation).
- Lisiecki, L. E., and M. E. Raymo (2005), A pliocene-pleistocene stack of 57 globally distributed benthic δ18O records, Paleoceanography , 20 (1).
- Tzedakis, P.C., Crucifix, M., Mitsui, T. and Wolff, E.W., A simple rule to determine which insolation cycles lead to interglacials. Nature, 542(7642), pp.427-432, February 2017.
- E. Widiasih, R. McGehee. A Quadratic Approximation to Budyko's Ice-Albedo Feedback Model with Ice Line Dynamics, *SIAM J. Appl. Dyn. Syst.*, March 2014.
- Walsh, Widiasih, Hahn, McGehee, Periodic orbits for a discontinuous vector field arising from a conceptual model of glacial cycles, Nonlinearity, May 2016.
- Wright H. and Stefanova I., Plant trash in the basal sediments of glacial lakes, Acta Palaeobotanica, 44 141–6, 2014

Thank you!