The Mathematics of a Hollywood Disaster Movie

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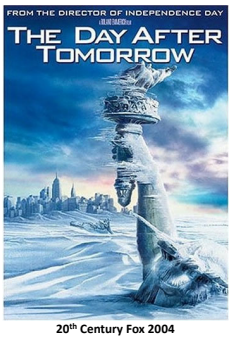
School of Mathematics
University of Minnesota

Mathematics of Climate Seminar
October 25, 2022

MCRN
Mathematics and Climate Research Network

Ocean Circulation



20th Century Fox 2004

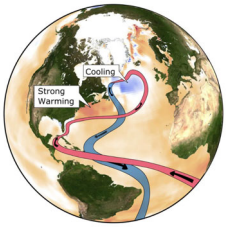
Ocean Circulation

The Washington Post

April 11, 2018

The oceans' circulation hasn't been this sluggish in 1,000 years. That's bad news.

The Atlantic Ocean circulation that carries warmth into the Northern Hemisphere's high latitudes is slowing down because of climate change, a team of scientists asserted Wednesday, suggesting one of the most feared consequences is already coming to pass.



Nature volume 556, pages191–196 (2018)

Ocean Circulation

Current Atlantic Meridional Overturning Circulation weakest in last millennium

The Atlantic Meridional Overturning Circulation (AMOC)—one of Earth's major ocean circulation systems—redistributes heat on our planet and has a major impact on climate. Here, we compare a variety of published proxy records to reconstruct the evolution of the AMOC since about AD 400. A fairly consistent picture of the AMOC emerges: **after a long and relatively stable period, there was an initial weakening starting in the nineteenth century, followed by a second, more rapid, decline in the mid-twentieth century, leading to the weakest state of the AMOC occurring in recent decades.**

NATURE GEOSCIENCE | VOL 14 | MARCH 2021 | 118–120 | www.nature.com/naturegeoscience118

Ocean Circulation

INDEPENDENT

April 12, 2018

Gulf Stream current at 'record low' with potentially devastating consequences for weather, warn scientists

The Atlantic meridional overturning circulation (AMOC), the system of currents that transports warm water from the tropics via the Gulf Stream to the North Atlantic, plays a major role in regulating the world's climate.

A fictional depiction of AMOC's collapse was portrayed in *The Day After Tomorrow*, and while the film's events were exaggerated, scientists say severe weather events are likely to result from the ongoing changes.

Ocean Circulation

The Day After Tomorrow/Film synopsis

After climatologist Jack Hall (Dennis Quaid) is largely ignored by U.N. officials when presenting his environmental concerns, his research proves true when an enormous "superstorm" develops, setting off catastrophic natural disasters throughout the world. Trying to get to his son, Sam (Jake Gyllenhaal), who is trapped in New York with his friend Laura (Emmy Rossum) and others, Jack and his crew must travel by foot from Philadelphia, braving the elements, to get to Sam before it's too late.

`Google Search`

Ocean Circulation

Dansgaard-Oeschger Events

"Global warming" can cause the Northern Hemisphere to cool.

Melting ice can lower the salinity of the North Atlantic, causing a decrease in the flow of the Atlantic Meridional Overturning Circulation (AMOC), slowing the heat transfer to the Northern Hemisphere.

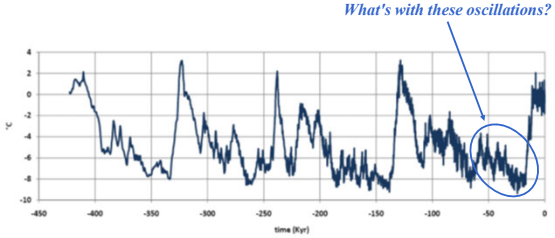
This phenomenon is believed to have caused the Younger Dryas.

Ocean Circulation

Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data

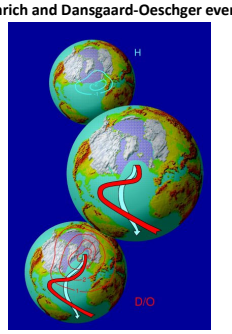
What's with these oscillations?



J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* 399, 429-436.

Ocean Circulation

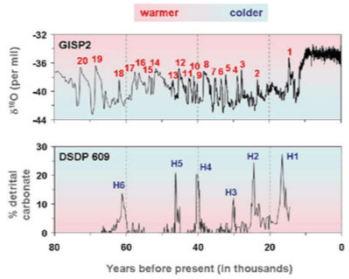
Heinrich and Dansgaard-Oeschger events



http://www.pik-potsdam.de/~stefan/sampleimages.html

Ocean Circulation

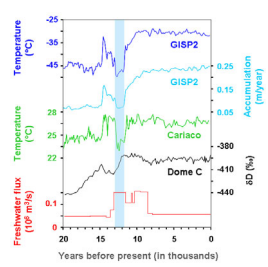
Heinrich and Dansgaard-Oeschger events



http://www.ncdc.noaa.gov/paleo/abrupt/data3.html

Ocean Circulation

The Younger Dryas



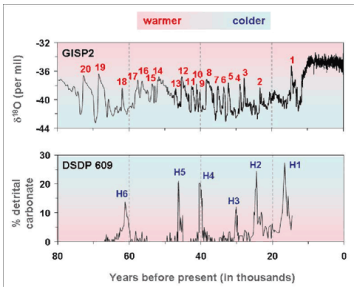
http://www.ncdc.noaa.gov/paleo/abrupt/data4.html

Mountain Avens
(*Dryas octopetala*)

https://www.ncdc.noaa.gov/abrupt-climate-change/The%20Younger%20Dryas

Ocean Circulation

Heinrich and Dansgaard-Oeschger events



What caused the oscillations?

Ocean Circulation

What caused the D-O oscillations?

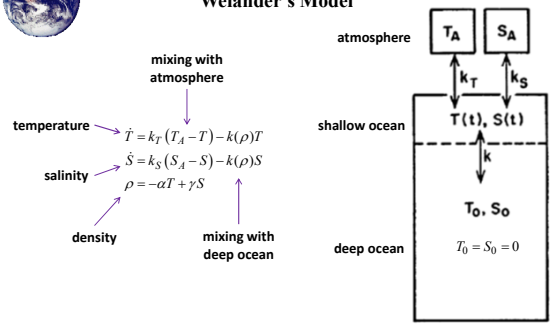
They could be self-oscillations in the natural dynamics of ocean circulation.

Pierre Welander, A simple heat-salt oscillator, *Dynamics of Atmospheres and Oceans* **6** (1982) 233-242.

R/V WeeLander is a 23-foot-long Beach Master work boat, informally named in honor of Professor Pierre Welander (1925—1996).

Ocean Circulation

Welander's Model



Pierre Welander, *Dynamics of Atmospheres and Oceans* 6 (1982).

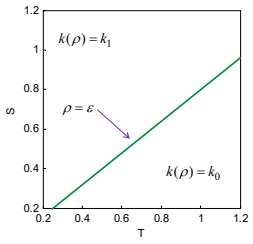
Ocean Circulation

Welander's Model

$$\dot{T} = k_T(T_A - T) - k(\rho)T$$
$$\dot{S} = k_S(S_A - S) - k(\rho)S$$
$$\rho = -\alpha T + \gamma S$$

The function k

$$k(\rho) = \begin{cases} k_0, & \rho < \varepsilon \\ k_1, & \rho > \varepsilon \end{cases}$$



Ocean Circulation

Welander's Model

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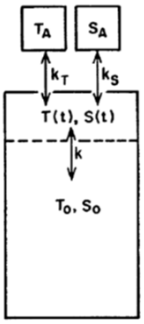
Welander chose scientifically reasonable values and dimensionless variables and constants

$$\dot{T} = 1 - T - k(\rho)T$$
$$\dot{S} = \beta(1 - S) - k(\rho)S$$
$$\rho = -\alpha T + S$$

$$k(\rho) = \begin{cases} 0, & \rho < \varepsilon \\ 1, & \rho > \varepsilon \end{cases}$$

$$\alpha = 0.8$$
$$\beta = 0.5$$



Ocean Circulation

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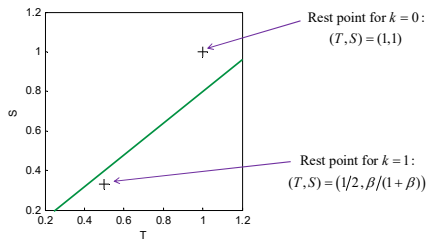

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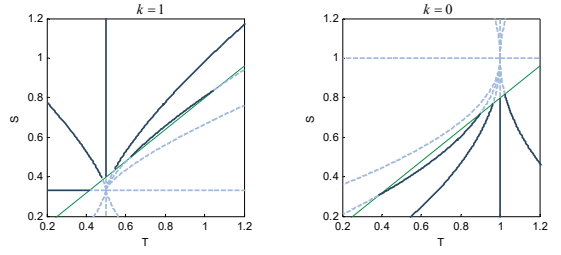

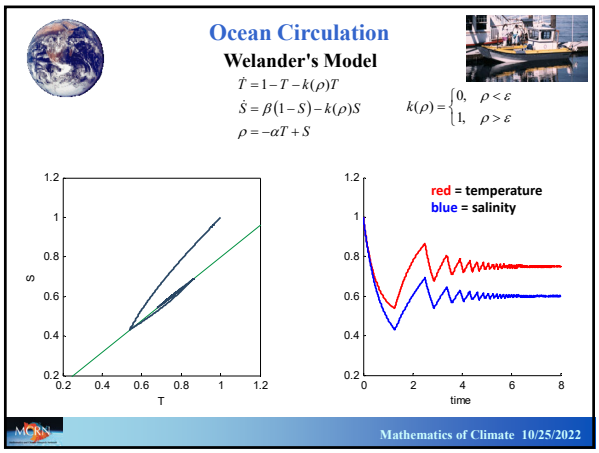
Ocean Circulation
Welander's Model
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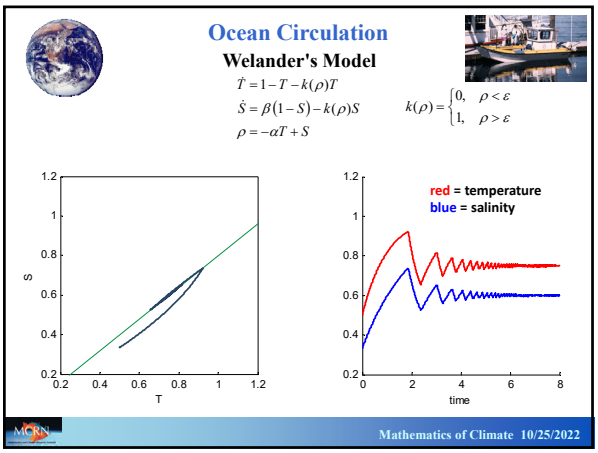
$k(\rho) = \begin{cases} 0, & \rho < \varepsilon \\ 1, & \rho > \varepsilon \end{cases}$
red = temperature
blue = salinity
S
T
time
Mathematics of Climate 10/25/2022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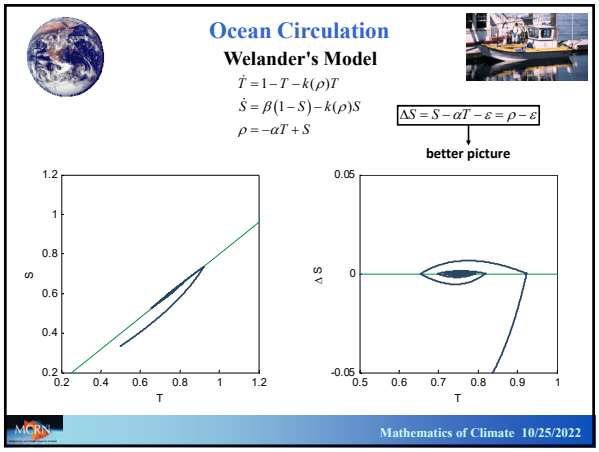
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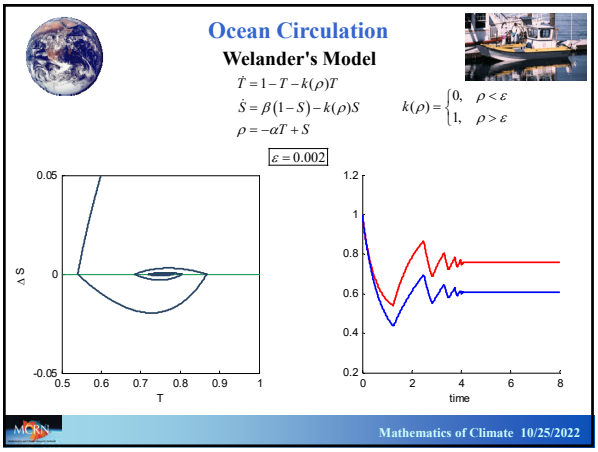
$\Delta S = S - \alpha T - \varepsilon = \rho - \varepsilon$
better picture
S
T
ΔS
Mathematics of Climate 10/25/2022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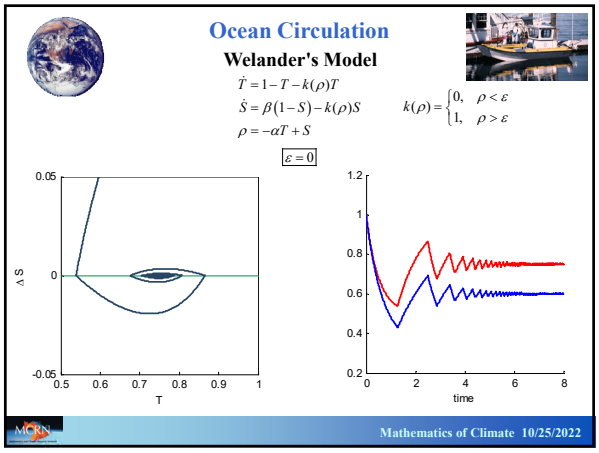
$\varepsilon = 0.002$
ΔS
T
time
Mathematics of Climate 10/25/2022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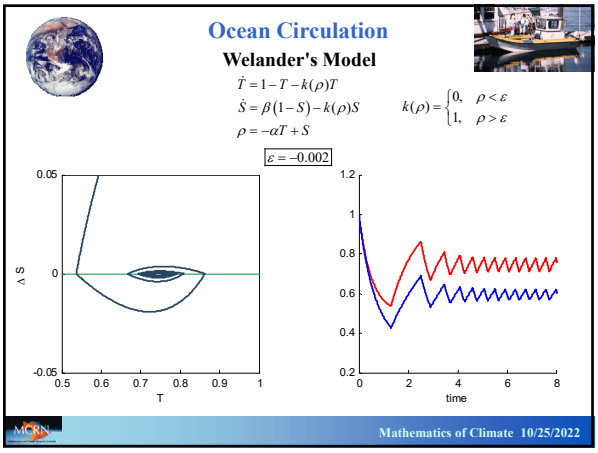
$\varepsilon = 0$
ΔS
T
time
Mathematics of Climate 10/25/2022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Mathematics of Climate 10/25/2022

Ocean Circulation

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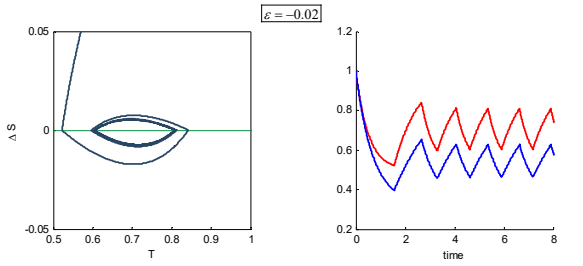
$\varepsilon = -0.02$



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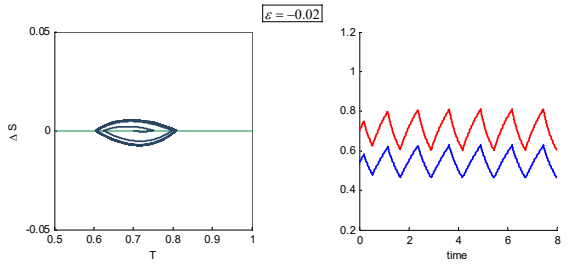

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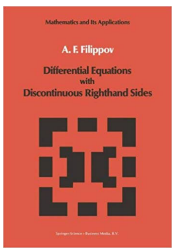
Ocean Circulation

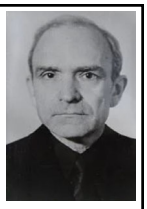
Filippov Approach



Kluwer Academic Publishers

Roughly

- The Euclidean space is partitioned by a finite number of sets.
- The boundaries are codimension 1.
- The vector field can be thought of as a finite number of vector fields, each defined and smooth on a partition set, including the boundary.
- The individual vector fields take different values on the boundaries.



A.F. Filippov*

*https://alchetron.com/Aleksei-Fedorovich-Filippov

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Ocean Circulation

Welander's Model

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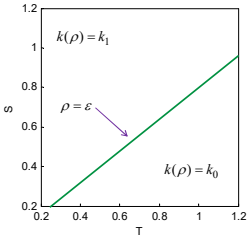
Welander's model is a Filippov system.



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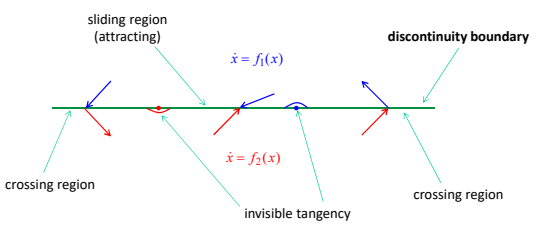
Ocean Circulation

Filippov Approach



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Ocean Circulation

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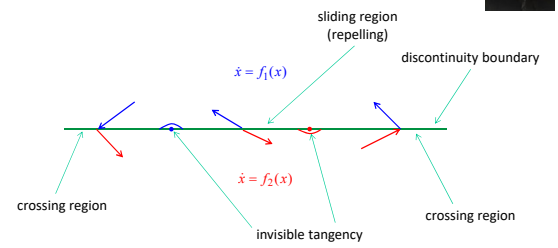

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Ocean Circulation

Welander Model

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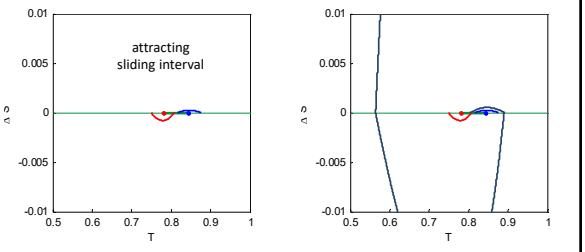
$\varepsilon = 0.025$



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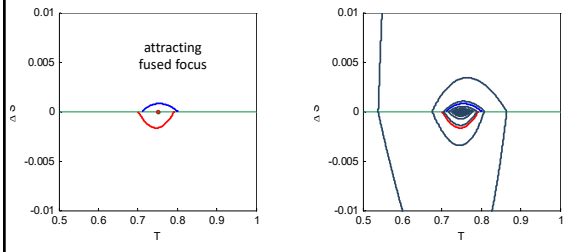

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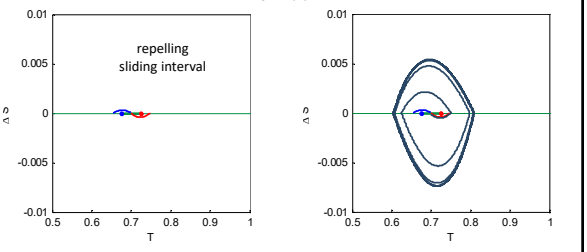

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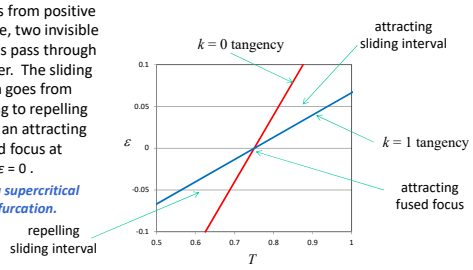
As ε goes from positive to negative, two invisible tangencies pass through each other. The sliding region goes from attracting to repelling through an attracting fused focus at $\varepsilon = 0$.

Looks like a supercritical Hopf bifurcation.



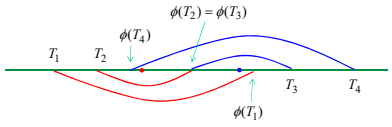
Ocean Circulation

Section Map

Follow the solution from when it crosses the discontinuity line until the next time it crosses the line.

$$T \mapsto \phi(T)$$

Not 1-1 for an attracting sliding region

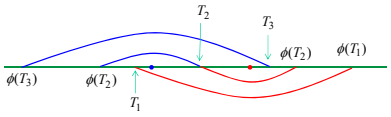


Ocean Circulation

Section Map

$$T \mapsto \phi(T)$$

Multi-valued for an repelling sliding region



Ocean Circulation
Welander Model

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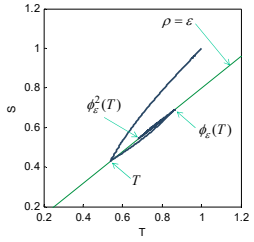
Follow the solution from when it crosses the discontinuity line ($\rho = \varepsilon$) until the next time it crosses the line. Use T as a coordinate on the line.

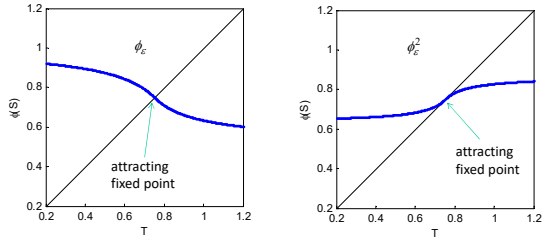
$$T \mapsto \phi_\varepsilon(T)$$



Ocean Circulation
Welander Model
Section Map

$$T \mapsto \phi_\varepsilon(T)$$

$\varepsilon = 0$



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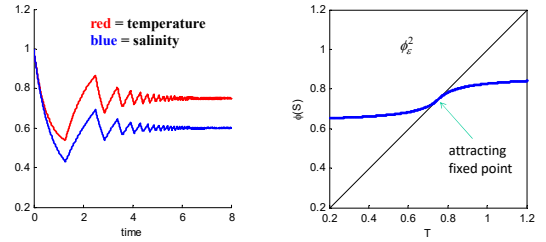
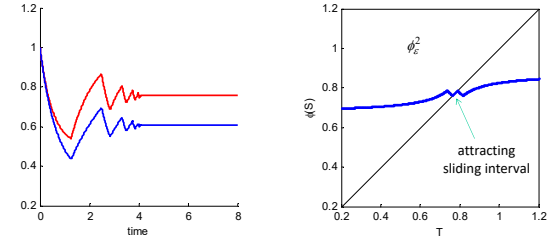


Ocean Circulation
Welander Model
Section Map

$$T \mapsto \phi_\varepsilon(T)$$

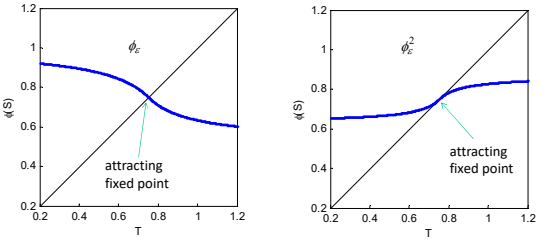
$\varepsilon = 0.01$



Ocean Circulation
Welander Model
Section Map

$$T \mapsto \phi_\varepsilon(T)$$

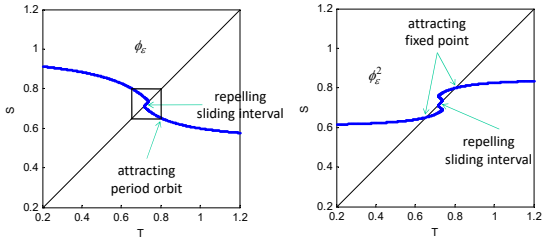
$\varepsilon = 0$



Ocean Circulation
Welander Model
Section Map

$$T \mapsto \phi_\varepsilon(T)$$

$\varepsilon = -0.01$



Ocean Circulation

Welander Model

Section Map

$T \mapsto \phi_\varepsilon(T)$

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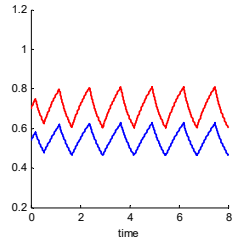
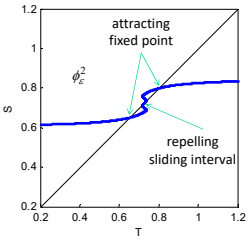




Ocean Circulation

Welander's Model

$\dot{T} = k_T(T_A - T) - k(\rho)T$

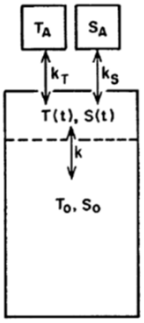
$\dot{S} = k_S(S_A - S) - k(\rho)S$

$\rho = -\alpha T + \gamma S$

Welander assumed that the self-oscillations he found in his discontinuous model would hold held for a nearby smooth system.

Juliann Leifeld, PhD 2016:

Welander's assumption was correct.

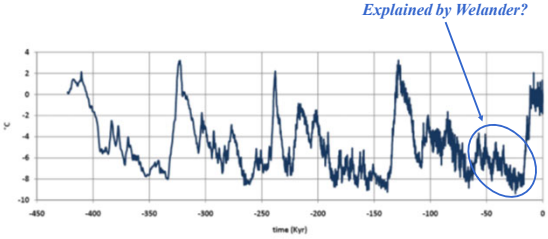


Ocean Circulation

Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data

Explained by Welander?

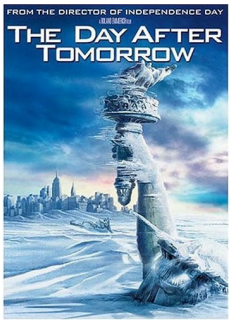


J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* 399, 429-436.

Ocean Circulation

Moral

Surprisingly, the moral of the film was NOT that everyone should study the Welander model.

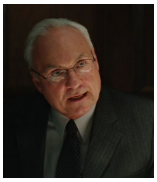


The Day After Tomorrow

After Los Angeles and Tokoyo were decimated, and as a tidal wave was about to inundate Manhattan, a conversation ensued between the Vice President of the United States and the leading paleoclimatologist.

The Day After Tomorrow

Moral



Vice President: ***"Maybe you should stick to science and leave policy to us."***

Scientist: ***"Well, we tried that approach. You didn't want to hear about the science when it could have made a difference."***