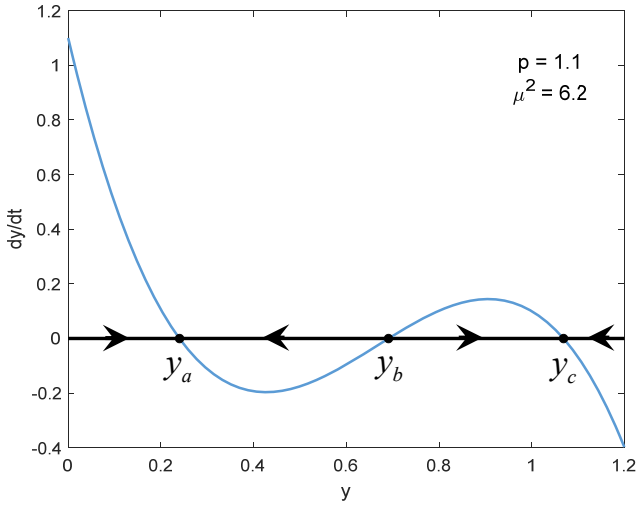
Homework Set 5

Due November 9, 2023

Please refer to the slides from the lecture on October 24 discussing Cessi's model. The equation and one of the figures are repeated here for your convenience. For the exercises, assume that $\mu^2 = 6.2$.

$$\frac{dy}{dt} = -\left(1 + \mu^2 (y-1)^2\right) y + p$$



1. Compute (to four significant figures) the value of p_1 satisfying the property:

   $1.1 < p < p_1 \Rightarrow$ the differential equation has exactly two stable rest points
   $p_1 < p \Rightarrow$ the differential equation has exactly one stable rest point

   Hint: p_1 occurs when the two rest points y_a and y_b in the figure have collided to become one rest point.

2. Compute (to four significant figures) the value of p_2 satisfying the property:

   $p_2 < p < 1.1 \Rightarrow$ the differential equation has exactly two stable rest points
   $p < p_2 \Rightarrow$ the differential equation has exactly one stable rest point

3. Suppose the system starts at the equilibrium y_a when $p = 1.1$. The value of p is raised to the value 1.4 and left there long enough for y to become close to the only stable equilibrium. Discuss how to change the value of p to return the system to a value of y close to y_a.