Set 3

For these exercises, assume that the Earth's obliquity is a constant 23.5°. Assume that the semimajor axis of the Earth's elliptical orbit around the Sun remains constant at 1.5×10^{11} m. Assume that the solar flux at this distance is $1368\ \mathrm{Wm}^{-2}$.

For Exercise 4, assume that the annual average solar flux F is

$$F = \frac{F_0}{\sqrt{1-e^2}} s(y),$$

where $F_0 = 1368\ \mathrm{Wm}^{-2}$, e is the eccentricity, and $s(y)$ is the distribution of insolation as a function of the sine of the latitude. Use Chylek and Coakley's quadratic approximation for s:

$$s(y) = 1 - 0.241(3y^2 - 1)\ .$$

1. Compute the maximum and minimum distances from the Sun to the Earth for eccentricities of $e = 0$, $e = 0.016$, $e = 0.03$, and $e = 0.06$. Compute the solar flux at these distances.

Solution. If a is the semimajor axis and e is the eccentricity, then the minimum distance is $a(1-e)$, while the maximum distance is $a(1+e)$. The flux at these distances is $\frac{F_0}{(1-e)^2}$ and $\frac{F_0}{(1+e)^2}$, respectively. We therefore have the following table.

e	$r_{\min}$	$r_{\max}$	$F_{\max}$	$F_{\min}$
0	1.50×10^{11}m	1.50×10^{11}m	1368 Wm^{-2}	1368 Wm^{-2}
0.016	1.476×10^{11}m	1.524×10^{11}m	1413 Wm^{-2}	1325 Wm^{-2}
0.03	1.455×10^{11}m	1.545×10^{11}m	1454 Wm^{-2}	1289 Wm^{-2}
0.06	1.41×10^{11}m	1.59×10^{11}m	1548 Wm^{-2}	1218 Wm^{-2}

2. Assume that the northern hemisphere summer solstice occurs at perihelion (the point on the orbit of minimum distance to the Sun). For each of the eccentricities in Exercise 1, compute the insolation at noon on the summer solstice for latitudes of 0° (the Equator), 23.5ºN (the Tropic of Cancer), 45°N (the latitude of the Twin Cities, 66.5ºN (the Arctic Circle), and 90°N (the North Pole).

Solution. At noon on an equinox, the sun is at an angle of θ from directly overhead, where θ is the latitude. On the summer solstice, the sun is at an angle of $\theta-\beta$ from directly overhead, where $\beta = 23.5^{\circ}$ is the obliquity. Therefore, the insolation is $F\cos(\theta-\beta)$, where

F is the solar flux, as shown in the following table, computed with $F = F_{\max}$ from the first exercise.

	latitude				
e	0	23.5°	45°	66.5°	90°
0	1255 Wm^{-2}	1368 Wm^{-2}	1273 Wm^{-2}	1000 Wm^{-2}	545 Wm^{-2}
0.016	1296 Wm^{-2}	1413 Wm^{-2}	1315 Wm^{-2}	1033 Wm^{-2}	563 Wm^{-2}
0.03	1333 Wm^{-2}	1454 Wm^{-2}	1353 Wm^{-2}	1063 Wm^{-2}	580 Wm^{-2}
0.06	1420 Wm^{-2}	1548 Wm^{-2}	1440 Wm^{-2}	1132 Wm^{-2}	617 Wm^{-2}

3. Repeat Exercise 2 with the assumption that the northern hemisphere summer solstice occurs at aphelion (the point on the orbit of maximum distance to the Sun).

Solution. This is the same as Exercise 2, but with $F = F_{\min}$.

	latitude				
e	0	23.5°	45°	66.5°	90°
0	1255 Wm^{-2}	1368 Wm^{-2}	1273 Wm^{-2}	1000 Wm^{-2}	545 Wm^{-2}
0.016	1215 Wm^{-2}	1325 Wm^{-2}	1233 Wm^{-2}	969 Wm^{-2}	528 Wm^{-2}
0.03	1183 Wm^{-2}	1289 Wm^{-2}	1200 Wm^{-2}	943 Wm^{-2}	514 Wm^{-2}
0.06	1117 Wm^{-2}	1218 Wm^{-2}	1133 Wm^{-2}	890 Wm^{-2}	485 Wm^{-2}

4. For each of the eccentricities in Exercise 1 and each of the latitudes in Exercise 2, compute the annual average insolation.

Solution. Plugging the values into the formula (the correct formula, with annual average insolation at $342\ \mathrm{Wm}^{-2}$, we produce the following table.

	latitude				
e	0	23.5°	45°	66.5°	90°
0	424.4 Wm^{-2}	385.1 Wm^{-2}	300.8 Wm^{-2}	216.5 Wm^{-2}	177.2 Wm^{-2}
0.016	424.5 Wm^{-2}	385.2 Wm^{-2}	300.8 Wm^{-2}	216.5 Wm^{-2}	177.2 Wm^{-2}
0.03	424.6 Wm^{-2}	385.3 Wm^{-2}	300.9 Wm^{-2}	216.6 Wm^{-2}	177.2 Wm^{-2}
0.06	425.2 Wm^{-2}	385.8 Wm^{-2}	301.3 Wm^{-2}	216.9 Wm^{-2}	177.5 Wm^{-2}

5. Compare and discuss your answers to Exercises 2, 3, and 4.

Solution. The maximum insolation at summer solstice for every latitude is much more sensitive to eccentricity than the annual average insolation.