Set 4 Solutions

Table S1. Planetary Heat Storage: Ocean, Ice, Air and Land.

Energy required to melt ice and warm the air, land and ocean by specified amounts.[1]

Ocean warming by 1°C through 1 km depth of ocean. Heat storage is 1°C × 10^5 g/cm^2 × 1 cal/g × 4.19 joules/cal × area Earth × 0.7 ~ 15 × 10^{23} joules ~ **93 W yr/m^2**.

Ice sheet melting to raise sea level 1 meter. Assume ice starts at –10°C and ends at mean ocean surface temperature (+15°C). Energy required is 100 cal/g (80 cal/g for melting). Energy for 1 meter of sea level: 100g/cm^2 × 100cal/g × 4.19 joules/cal × area Earth × 0.7 ~1.5 × 10^{23} joules ~ **9.3 W yr/m^2**.

Sea ice melting (all sea ice on planet). Assume ice starts at –10°C and ends at mean ocean surface temperature (+15°C), and that sea ice covers 4% of the planet with mean thickness 2.5 m. Energy required is 250 g cm^2 × 100 cal/g (80 cal/g for melting) × 4.19 joules/cal × 0.04 × area Earth ~ 2.14 × 10^{22} joules ~ **1.3 W yr/m^2**.

Air warming by 1°C. The Earth's atmospheric mass is ~ 10 m of water. Heat capacity of air ~ 0.24 cal/g/°C. Energy to raise air temperature 1°C: 1°C × 1000 g/cm^2 × 0.24 cal/g/°C × 4.19 joules/cal × area Earth ~ 0.26 ×10^{22} joules ~ **0.32 W yr/m^2.**

Land surface warming by 1°C: The depth of penetration of a thermal wave into the Earth's crust in 10 years, weighted by ΔT, is ~10 m. With density ~ 3 g/cm^3, heat capacity ~ 0.2 cal/g/°C, and 0.29 fractional land coverage, land heat storage is 10^3 cm ×3 g/cm^3 × 0.2 cal/g/°C × 1°C ×4.19 joules/cal × area Earth × 0.29 ~ 0.37 × 10^{22} joules ~ **0.23 W yr**. [In a century the depth of penetration is ~3 times more than in a decade, so heat storage in a century due to 1°C warming is ~ **0.7 W yr/m^2**.]

[1]Note that 1 W sec = 1 joule, # sec/year ~ π×10^7, area Earth ~ 5.1×10^{18} cm^2, 1 W yr over full Earth ~ 1.61×10^{22} joules, ocean fraction of Earth ~ 0.7, 1 calorie ~ 4.19 joules.

1. Assuming an energy imbalance of 1.2 W/m^2, how long would it take to raise the temperature of the entire ocean by 5°C? (You may assume that the average ocean depth is 4300 meters). How long would it take to melt enough ice to raise the ocean depth by 70 meters, leaving the ocean temperature unchanged?

Solution. It takes 93 Wyr/m^2 to raise 1 km by 1°C. Therefore, $93\times4.3\times5 \approx 2000 \text{ Wyr/m}^2$ will raise 4.3 km by 5°C. With a heat imbalance of 1.2 Wyr/m^2, it would take $2000/1.2 \approx \boxed{1700 \text{ years}}$.

It takes 9.3 Wyr/m^2 to raise the sea level by 1 meter. Therefore, $9.3\times70 = 651 \text{ Wyr/m}^2$ will raise the sea level by 70 meters. With a heat imbalance of 1.2 Wyr/m^2, it would take $651/1.2 \approx \boxed{540 \text{ years}}$.

2. By some estimates, it took 20,000 years to raise the temperature of the entire ocean 5°C during the PETM. What energy imbalance would account for that rise? (Assume all other temperatures were unchanged and that the ocean depth was 4400 meters.)

Solution. It takes $93\ \text{Wyr/m}^2$ to raise $1\ \text{km}$ by 1°C. With an ocean depth of 4400 m, it would take $93 \times 4.4 \times 5 = 2046\ \text{Wyr/m}^2$ to raise the ocean temperature by 5°C. If it takes $20,000$ years, the heat imbalance would have to average $2046/20000 = \boxed{0.1023\ \text{W/m}^2}$.

3. What energy imbalance would be required to melt enough ice to raise the ocean depth by 70 meters in 1000 years, assuming all atmosphere, land, and ocean temperatures remained constant? What energy imbalance would accomplish the same rise in 100 years?

Solution. According to our computation in Exercise 1, it takes $651\ \text{Wyr/m}^2$ to raise the sea level by 70 meters. To accomplish this rise in 1000 years would take a average imbalance of $651/1000 = \boxed{0.651\ \text{W/m}^2}$. To accomplish it in 100 years would take an average imbalance of $651/100 = \boxed{6.51\ \text{W/m}^2}$.

4. Assume that, over the course of 100,000 years, the air and land temperatures fell by 5°C, as did the top kilometer of the ocean, and enough glaciers formed to lower the sea level by 125 meters. What average energy imbalance would be required?

Solution. First let's compute how much energy it would take to accomplish those changes in one year.

Air temperature: It takes $0.32\ \text{Wyr/m}^2$ to raise the air temperature by 1°C. Therefore, $0.32 \times (-5) = -1.6\ \text{Wyr/m}^2$ will raise the air temperature by -5°C, i.e., lower it by 5°C.

Land temperature: It takes $0.23\ \text{Wyr/m}^2$ to raise the land surface temperature by 1°C. Raising the temperature by -5°C therefore takes an imbalance of $0.23 \times (-5) = -1.15\ \text{Wyr/m}^2$.

Ocean temperature: It takes $93\ \text{Wyr/m}^2$ to raise $1\ \text{km}$ by 1°C. Therefore, $93 \times (-5) = -465\ \text{Wyr/m}^2$ will raise the temperature by -5°C.

Glacier formation: It takes $9.3\ \text{Wyr/m}^2$ to raise the sea level by 1 meter through melting glaciers. Therefore, it takes $9.3 \times (-125) = -1162.5\ \text{Wyr/m}^2$ to raise the sea level by -125 meters through melting glaciers. Turning this around, it would take an imbalance of $-1162.5\ \text{Wyr/m}^2$ to form enough ice to lower the sea level by 125 meters.

Altogether: Adding up all the imbalances, to accomplish all these feats takes $-1.6 - 1.15 - 465 - 1162.5 \approx -1630\ \text{Wyr/m}^2$. Over the course of 100,000 years, the average energy imbalance would be $-1630/100000 \approx \boxed{-0.163\ \text{Wyr/m}^2}$.

Note: Under *Land surface warming*, Hansen points out that one needs more energy to heat the land surface over a longer period of time, since it takes time for the heat to penetrate deeper. The figure he gives for a decade is 0.23. He estimates that a century would take three times as much heat. That would change the 1.15 computed above to 3.45, but it would not change the answer in the first three significant figures. The actual number for

100,000 years would have to take into consideration that the depth of penetration also depends on the heat flow from the interior of the planet, which would be beyond the scope of this class.

5. (*The Day After Tomorrow* scenario). Assume that, over the course of 6 weeks, the air and land temperature dropped by 10°C, as did the top 100 meters of the ocean, and enough snow accumulated on land to lower the sea level by 2 meters. What energy imbalance would be required?

Solution. As for Exercise 4, we first compute how much energy it would take to accomplish those changes in one year.

Air temperature: It takes $0.32\ \text{Wyr/m}^2$ to raise the air temperature by 1°C. Therefore, $0.32\times(-10) = -3.2\ \text{Wyr/m}^2$ will lower the air temperature by 10°C.

Land temperature: Lowering the land temperature in 6 weeks would take a negligible amount of energy, but we can use Hansen's number of $0.23\ \text{Wyr/m}^2$ for a 1°C increase. Therefore, $0.23\times(-10) = -2.3\ \text{Wyr/m}^2$ will lower the land temperature by 10°C.

Ocean temperature: It takes $93\ \text{Wyr/m}^2$ to raise $1\ \text{km}$ by 1ºC. Therefore, it takes $93\times 0.1\times(-10) = -93\ \text{Wyr/m}^2$ to lower the temperature by 10°C.

Snow accumulation: It takes $9.3\ \text{Wyr/m}^2$ to raise the sea level by 1 meter through melting glaciers. Therefore, it takes $9.3\times(-2) = -18.6\ \text{Wyr/m}^2$ to lower the sea level by 2 meters through accumulating snow.

Altogether: Adding up all the imbalances, to accomplish The Day After Tomorrow scenario takes $-3.2-2.3-93-18.6 \approx -117.1\ \text{Wyr/m}^2$. Six weeks is 42 days, or $42/365 = 0.115$ years, so the heat imbalance would be $-117.1/0.115 \approx \boxed{1020\ \text{Wyr/m}^2}$.

Note: As mentioned above, the amount of heat released from the land over six weeks would be negligible compared to that of 10 years in Hansen's computation. If we ignore the land, the arithmetic becomes $-3.2-93-18.6 \approx -114.8\ \text{Wyr/m}^2$, or a heat imbalance of $-114.8/0.115 \approx 998\ \text{Wyr/m}^2$. In either case, the Earth would have to radiate energy into space at almost three times the rate at which it is coming from the Sun, which could only happen if the photosphere became really hot. No explanation for how that could happen was given in the film.