CORRECTIONS TO FIRST PRINTING OF

Olver, P.J., *Equivalence, Invariants, and Symmetry*,
Cambridge University Press, Cambridge, 1995.

Last modified: January 4, 2021

⋆⋆⋆ *On back cover, line 17–18, change*

prospective geometry

to

projective geometry

⋆⋆⋆ *page xv, add to acknowledgements*

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⋆⋆⋆ *page 22, Theorem 1.28, line 3, change*

... all $t, s \in \mathbb{R}$ where the equation is defined.

to

... all $t, s \in V$ where $V \subset \mathbb{R}^2$ is a connected open subset of the (t, s) plane containing $(0, 0)$ consisting of points where the equation is defined.

⋆⋆⋆ *page 32, line 12-13, change*

an (necessarily unique)

to

a (necessarily unique)

⋆⋆⋆ *page 32, line before Definition 2.1, change*

stucture

to

structure

⋆⋆⋆ *page 36, line before Example 2.9, change*

$\mathrm{GL}(2)$

to

$\mathrm{GL}(2, \mathbb{C})$.

⋆⋆⋆ *page 39, Example 2.13, change the first two occurrences of*

$\mathrm{PSL}(n, \mathbb{R})$

to

$\mathrm{PGL}(n, \mathbb{R})$.

$\star\star\star$ *Also append to the last sentence*

$\mathrm{PSL}(n,\mathbb{R}) = \mathrm{SL}(n,\mathbb{R})/\{\pm \mathbb{1}\}$ is equal to the connected component of $\mathrm{PGL}(n,\mathbb{R})$ containing the identity.

$\star\star\star$ *page 51, equation* (2.14), *change*

$$C^k_{ij} = -C^k_{ij}$$

to

$$C^k_{ji} = -C^k_{ij}$$

$\star\star\star$ *page 55, lines 4–5, change*

$G_H = \{g \mid gHg^{-1} \subset H\}$ has Lie algebra

to

$G_H = \{g \mid gHg^{-1} \subset H\}$ is a normal subgroup with Lie algebra

$\star\star\star$ *page 61, line 31, change*

there is a scalar function $h_{\mathbf{v}}(t)$ such that

to

there is a function $h_{\mathbf{v}}\colon \mathbb{R}^k \to \mathbb{R}^k$ such that

$\star\star\star$ *page 65, Example 2.80, line 8, change*

$$\mathbf{v}(HF) = 0$$

to

$$\mathbf{v}(H) = 0.$$

$\star\star\star$ *page 73, line 9, change*

$$\begin{pmatrix} a^{-1}\,da & a^{-1}(a\,db - b\,da) \\ 0 & 1 \end{pmatrix}$$

to

$$\begin{pmatrix} a^{-1}\,da & a^{-1}\,db \\ 0 & 0 \end{pmatrix}$$

$\star\star\star$ *page 85, equation* (3.18), *change*

$$1 + t h_{\mathbf{v}}(x) + \tfrac{1}{2} t^2 \mathbf{v}(h_{\mathbf{v}}) + \cdots$$

to

$$1 + t h_{\mathbf{v}}(x) + \tfrac{1}{2} t^2 [\mathbf{v}(h_{\mathbf{v}}) + h_{\mathbf{v}}^2] + \cdots$$

$\star\star\star$ *page 87, equation* (3.21), *change*

$$\sigma([\mathbf{v},\mathbf{w}]) = \widehat{\mathbf{w}}(\sigma(\mathbf{v})) - \widehat{\mathbf{v}}(\sigma(\mathbf{w}))$$

to

$$\sigma([\mathbf{v},\mathbf{w}]) = \widehat{\mathbf{v}}(\sigma(\mathbf{w})) - \widehat{\mathbf{w}}(\sigma(\mathbf{v}))$$

⋆⋆⋆ *page 93, change the first full paragraph*

In order to formulate a general theorem governing ... constructed in this manner.

to

In order to formulate a general theorem governing the existence of relative invariants for sufficiently regular group actions, we consider the extended group action (3.15) on the bundle $E = M \times U$ and its dual version $(x, v) \mapsto (g \cdot x, \mu(g, x)^{-T})$ on the dual bundle $E^* = X \times U^*$. The key remark is that there is a one-to-one correspondence between relative invariants of weight μ and linear absolute invariants of the dual action. Specifically, a linear function $J(x, v) = \sum_{\alpha=1}^{n} R_\alpha(x) v^\alpha$ is an invariant of the dual action on E^* if and only if the vector-valued function $R(x) = (R_1(x), \ldots, R_q(x))^T$ is a relative invariant of weight μ. Therefore, we need only produce a sufficient number of *linear* invariants of the extended action. Moreover, if $J(x, v)$ is any invariant of the extended group action, then it is not hard to prove that its linear Taylor polynomial is also an invariant, and hence provides a relative invariant for the multiplier representation. Thus, the only question is how many independent relative invariants can be constructed in this manner.

⋆⋆⋆ *page 94, lines 26–28, change*

I do not know a general theorem that counts the number of relative invariants of multiplier representations that do not satisfy the hypotheses of Theorem 3.36

to

A general theorem that counts the number of relative invariants of multiplier representations in all cases can be found in the recent paper by M. Fels and the author, "On relative invariants", *Math. Ann.* **308** (1997), 701–732.

⋆⋆⋆ *page 96, equation* (3.30), *change*

$$\begin{aligned}
\mathbf{v}_- &= a_1 \frac{\partial}{\partial a_0} + 2a_2 \frac{\partial}{\partial a_1} + \cdots + (n-1)a_{n-1} \frac{\partial}{\partial a_{n-2}} + n a_n \frac{\partial}{\partial a_{n-1}}, \\
\mathbf{v}_0 &= -n a_0 \frac{\partial}{\partial a_0} - (n-2)a_1 \frac{\partial}{\partial a_1} + \cdots + (n-2)a_{n-1} \frac{\partial}{\partial a_{n-1}} + n a_n \frac{\partial}{\partial a_n}, \\
\mathbf{v}_+ &= n a_0 \frac{\partial}{\partial a_1} + (n-1)a_1 \frac{\partial}{\partial a_2} + \cdots + 2a_{n-2} \frac{\partial}{\partial a_{n-1}} + a_{n-1} \frac{\partial}{\partial a_n}.
\end{aligned}$$

to

$$\begin{aligned}
\mathbf{v}_- &= n a_1 \frac{\partial}{\partial a_0} + (n-1)a_2 \frac{\partial}{\partial a_1} + \cdots + 2a_{n-1} \frac{\partial}{\partial a_{n-2}} + a_n \frac{\partial}{\partial a_{n-1}}, \\
\mathbf{v}_0 &= n a_0 \frac{\partial}{\partial a_0} + (n-2)a_1 \frac{\partial}{\partial a_1} + \cdots + (2-n)a_{n-1} \frac{\partial}{\partial a_{n-1}} - n a_n \frac{\partial}{\partial a_n}, \\
\mathbf{v}_+ &= a_0 \frac{\partial}{\partial a_1} + 2a_1 \frac{\partial}{\partial a_2} + \cdots + (n-1)a_{n-2} \frac{\partial}{\partial a_{n-1}} + n a_{n-1} \frac{\partial}{\partial a_n}.
\end{aligned}$$

$\star\star\star$ *page 108, line 24, change*

$\cot\theta \neq a$

to

$\cot t \neq a$

$\star\star\star$ *page 110, Theorem 4.6, line 2, change*

r-dimensional orbits

to

s-dimensional orbits

$\star\star\star$ *page 113, line 7, change*

$\bar{z}_0 = (\bar{x}_0, \bar{u}_0^{(n)}) = (x_0, \bar{f}(x_0))$

to

$\bar{z}_0 = (\bar{x}_0, \bar{u}_0^{(n)}) = (x_0, \bar{f}^{(n)}(x_0))$

$\star\star\star$ *page 119, equation* (4.31), *change*

$$\sum_{\#J \geq 0}$$

to

$$\sum_{\#J=0}^{n}$$

$\star\star\star$ *page 119, equation* (4.32), *change*

$D_i.$

to

$D_i^{(n)},$

and add the following sentence:

where $D_i^{(n)}$ denotes the order n truncation of the i^{th} total derivative, i.e., the summation in (4.18) is just over $0 \leq \#J \leq n$.

$\star\star\star$ *page 120, second line after equation* (4.35), *change*

The Lie algebra (4.14)

to

The Lie algebra (4.35)

$\star\star\star$ *page 124, first displayed equation, add subscript $_i$ to Q in first summation*

$$\omega = \sum_{i=1}^{p} Q_i(x, u^{(n)})\, dx^i + \sum_{\alpha=1}^{q} \sum_{\#J \leq n} P_\alpha^J(x, u^{(n)})\, du_J^\alpha$$

⋆⋆⋆ *page 126, line 12, change*

$$(\Psi^{(n)})^*\theta$$

to

$$\Psi^*\theta$$

⋆⋆⋆ *page 142, line 28, change*

$$s_0 = 1,\ s_1 = 2, \ldots, s_{r-3} = s_{r-2} = r-1$$

to

$$s_0 = 2,\ s_1 = 3, \ldots, s_{r-3} = s_{r-2} = r-1$$

⋆⋆⋆ *page 144, line 10, change*

$$a^\nu_\mu\, \xi^i_\nu$$

to

$$A^\nu_\mu\, \xi^i_\nu$$

⋆⋆⋆ *page 148, equation* (5.15)*, change*

$$\mathbf{v}_0 = x\,\frac{\partial}{\partial x} - \frac{n}{2} u\,\frac{\partial}{\partial u}\,, \qquad \mathbf{v}_+ = x^2\,\frac{\partial}{\partial x} - nxu\,\frac{\partial}{\partial u}\,.$$

to

$$\mathbf{v}_0 = x\,\frac{\partial}{\partial x} + \frac{n}{2} u\,\frac{\partial}{\partial u}\,, \qquad \mathbf{v}_+ = x^2\,\frac{\partial}{\partial x} + nxu\,\frac{\partial}{\partial u}\,.$$

⋆⋆⋆ *page 159, lines 5, 15 & 18, change*

$$d_{n+1}K_1 \wedge \cdots \wedge d_{n+1}K_r$$

to

$$d_{n+1}[\mathcal{D}K_1] \wedge \cdots \wedge d_{n+1}[\mathcal{D}K_r]$$

⋆⋆⋆ *page 171, lines 20 & -8, change*

$$n+2$$

to

$$n+1$$

⋆⋆⋆ *page 171, line -7 to -3, delete sentence*

Moreover, if the stable ... have order at most $n+1$.

⋆⋆⋆ *page 173, Example 5.52, line 2, after* "... via the standard representation"*, add*

$(x, y, u) \mapsto (\alpha x + \beta y, \gamma x + \delta y, u)$, where $\alpha\delta - \beta\gamma = 1$

⋆⋆⋆ *page 174, add remark that the referenced formula for the curvature that appears in* [**106**, p. 26] *is not quite correct. The denominator should be raised to the power* 3/2.

⋆⋆⋆ *page 188, line -2, change*

$\log x = h(u/x)$

to

$\log x = h(u/x^m)$

⋆⋆⋆ *page 190, lines 8–9, change*

H-reduced equationsymmetry reduced equation $\Delta/H = 0$ admits the corresponding normalizer subgroup $G_H = \{g | g \cdot H \cdot g^{-1} \subset H\}$ as a symmetry group.

to

H-reduced equation $\Delta/H = 0$ admits the quotient group G_H/H, where $G_H = \{g | g \cdot H \cdot g^{-1} \subset H\}$ is the normalizer subgroup, as a symmetry group.

⋆⋆⋆ *page 190, line 18, change*

$\eta \partial_y + \zeta \partial_u + \zeta^y \partial_{v_y}$

to

$\eta \partial_y + \zeta \partial_v + \zeta^y \partial_{v_y}$

⋆⋆⋆ *page 190, line 22, change*

$\mathbf{v} = \partial_y$

to

$\mathbf{v} = \partial_v$

⋆⋆⋆ *page 192, formula* (6.32), *change*

$(1 + u_x)^{3/2}$

to

$(1 + u_x^2)^{3/2}$

⋆⋆⋆ *page 192, displayed formula after* (6.32), *change*

$(1 + \theta_r^2)$

to

$(1 + r^2 \theta_r^2)^{3/2}$

⋆⋆⋆ *page 195, line -4, change*

Alternatively, $x = w_{uu}/w_u$, where w is an arbitrary solution ...

to

Alternatively, $w = x_{uu}/x_u$ is an arbitrary solution ...

⋆⋆⋆ *page 198, line 9, change*

$y = f(x)$

to

$w = f(x)$

$\star\star\star$ *page 198, equation* (6.56), *change*

y

to

w

$\star\star\star$ *page 201, equation* (6.61), *change*

$$\det \begin{vmatrix} \xi_1 & \varphi_1 & \varphi_1^1 & \cdots & \varphi_1^{r-1} \\ \xi_2 & \varphi_2 & \varphi_2^1 & \cdots & \varphi_2^{r-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \xi_r & \varphi_r & \varphi_r^1 & \cdots & \varphi_r^{r-1} \end{vmatrix} = 0.$$

to

$$\det \begin{vmatrix} \xi_1 & \varphi_1 & \varphi_1^1 & \cdots & \varphi_1^{r-2} \\ \xi_2 & \varphi_2 & \varphi_2^1 & \cdots & \varphi_2^{r-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \xi_r & \varphi_r & \varphi_r^1 & \cdots & \varphi_r^{r-2} \end{vmatrix} = 0.$$

$\star\star\star$ *page 213, equation* (6.84), *change all* p*'s to* f*'s*:

$$\eta(x) = \left| f^n(x) \right|^{(1-n)/(2n)} \, \exp\left\{ \int^x \frac{f^{n-1}(y)}{n f^n(y)} \, dy \right\}. \tag{6.84}$$

$\star\star\star$ *page 218, line -2, change*

$f^k(x) = W^k(x)$

to

$f^k(x) = (-1)^k \, W^k(x)$

$\star\star\star$ *page 226, line 6, change*

$P(t, x, u^{(2n)})$

to

$R(t, x, u^{(2n)})$

$\star\star\star$ *page 231, lines -4 & -1, change*

$E(\overline{L})$

to

$\overline{E}(\overline{L})$

$\star\star\star$ *page 238, Exercise 7.26, delete the sentence*

Determine the conservation laws associated with the point symmetries found in Exercise 6.16.

since the precise connection between symmetries and conservation laws has not been discussed in this book. (*See, however,* [**186**].)

$\star\star\star$ *page 240, in Remark, replace two sentences:*

However, I do not know ... I. Anderson, [**7**].

by

See the paper by I.A. Kogan and the author, "Invariant Euler–Lagrange equations and the invariant variational bicomplex", *Acta Appl. Math.* **76** (2003), 137–193, for a general formula for calculating the invariant formulation of the Euler–Lagrange equations directly from the invariant formula for the Lagrangian.

$\star\star\star$ *page 243, lines 18 & 20, change*

$(x, v_y, v_{yy}, \ldots)$

to

$(y, v_y, v_{yy}, \ldots)$

$\star\star\star$ *page 293, line 7, change*

$a_4 = 0$

to

$a_4 = a_5 = 0$

$\star\star\star$ *page 293, equations* (9.30) & (9.32), *change*

$\bar{a}_6 \omega^3 = a_6 \omega^3$

to

$\bar{a}_6 \overline{\omega}^3 = a_6 \omega^3$

$\star\star\star$ *page 307, line 13, change*

$\widetilde{\alpha}^\kappa = \sum_k z_j^\kappa(x)\, \theta^j$

to

$\widetilde{\alpha}^\kappa = \sum_j z_j^\kappa(x)\, \theta^j$

$\star\star\star$ *page 307, equation* (10.7), *change*

$$\sum_{k=1}^{r} z_j^\kappa\, \theta^j$$

to

$$\sum_{j=1}^{m} z_j^\kappa\, \theta^j$$

⋆⋆⋆ *page 309, equation* (10.12), *change*

$$\sum_{i=1}^{p} z_i^\kappa\,\theta^i$$

to

$$\sum_{i=1}^{m} z_i^\kappa\,\theta^i$$

⋆⋆⋆ *page 339, line 6, delete first*

arc length

⋆⋆⋆ *page 341, line -3, change*

I_4

to

I_5

⋆⋆⋆ *page 349, line -12, change*

$\alpha^1 - T^1_{12}\theta^1 \wedge \theta^2 - T^1_{13}\theta^1 \wedge \theta^3$

to

$\alpha^1 - T^1_{12}\theta^2 - T^1_{13}\theta^3$

⋆⋆⋆ *page 367, line 10, change*

manifolds M

to

manifolds M and $\overline{M}$

⋆⋆⋆ *page 368, equation* (11.30), *change*

$= T\,\omega^1 \wedge \omega^2 \wedge \omega^3 = T\Omega.$

to

$= T\,\omega^1 \wedge \omega^2 \wedge \omega^3.$

⋆⋆⋆ *page 372, lines 13–16, change*

However, I do not know any naturally occurring examples exhibiting this phenomenon, and, moreover, the prolongation procedure to be discussed below will handle this (remote) possibility as well.)

to

However, the prolongation procedure to be discussed below will handle this possibility as well; an example is the equivalence problem for a parabolic evolution equation analyzed in [**69**].)

$\star\star\star$ *page 375, line 5, change*

(12.3)

to

(12.1)

$\star\star\star$ *page 394, lines 16 & 21, change*

(11.6)

to

(11.7)

$\star\star\star$ *page 394, line 22, change*

vector S

to

matrix S

$\star\star\star$ *page 395, equation* (12.52), *change*

$$\varpi = \alpha + S\,\theta, \qquad \text{or explicitly,} \qquad \varpi^i = \alpha^i + \sum_{j=1}^{m} S^i_j \theta^j$$

to

$$\varpi = \alpha - S\,\theta, \qquad \text{or explicitly,} \qquad \varpi^i = \alpha^i - \sum_{j=1}^{m} S^i_j \theta^j$$

$\star\star\star$ *page 406, equation* (12.73), *change*

$$Q_p \widehat{D}_x Q_{pp} 6 Q_{uu}$$

to

$$Q_p \widehat{D}_x Q_{pp} + 6\,Q_{uu}$$

$\star\star\star$ *page 411, lines 12–13, change*

$$c(x,y,\varphi(x,y))\,\frac{\partial\varphi}{\partial x} = a(x,y,\varphi(x,y)),$$
$$c(x,y,\varphi(x,y))\,\frac{\partial\varphi}{\partial y} = b(x,y,\varphi(x,y)).$$

to

$$c(x,y,\varphi(x,y))\,\frac{\partial\varphi}{\partial x} = -a(x,y,\varphi(x,y)),$$
$$c(x,y,\varphi(x,y))\,\frac{\partial\varphi}{\partial y} = -b(x,y,\varphi(x,y)).$$

⋆⋆⋆ *page 423, equation* (14.4)*, change*

$\Phi(t, w)$

to

$\Phi(t, s)$

⋆⋆⋆ *page 425, lines 3–6, change*

There is, however, a four-parameter group action obtained by including the additional generator $z\partial_y$, whose associated one-parameter group $(x, y, z) \mapsto (x, y + \mu z, z)$ can be recovered from the previous group transformations by taking commutators.

to

Moreover, one cannot include these vector fields in a finite-dimensional Lie algebra, since $[\mathbf{v}_2, \mathbf{v}_3] = \mathbf{v}_4 = z\partial_y$, $[\mathbf{v}_4, \mathbf{v}_3] = \mathbf{v}_5 = z^2\partial_y$, and so on, hence the successive commutators span an infinite-dimensional Lie algebra of vector fields.

⋆⋆⋆ *pages 425, lines 33–34, change*

Relative invariants correspond to *linear* invariants $J(x, u) = R(x) \cdot u = \sum_{\alpha=1}^{q} R_\alpha(x) u^\alpha$ of the extended action, ...

to

Relative invariants of the dual action on $E^* = X \times U^*$ correspond to *linear* invariants $J(x, u) = \sum_{\alpha=1}^{q} R_\alpha(x) u^\alpha$ of the extended action, ...

⋆⋆⋆ *page 437, line -9, change*

... the rank zero case in Theorem 4.24.

to

... the rank zero case in Theorem 4.18.

⋆⋆⋆ *page 442, Figure 5, change*

L

to

M

⋆⋆⋆ *page 446, line 5, change*

... restrictions of $\boldsymbol{\theta}$ to U and V, so that

to

... restrictions of $\boldsymbol{\theta}$ to U and $\widetilde{U}$, so that

⋆⋆⋆ *page 472, Table 1, Case 1.8, column 3, change*

$\mathfrak{a}(1) \ltimes \mathbb{C}^k$

to

$\mathbb{C} \ltimes (\mathbb{C} \ltimes \mathbb{C}^k)$

⋆⋆⋆ *page 475, Table 6, Case 6.1:*

It would be better to replace α by β in this entry. A good exercise is to determine the relation between α and β in the complex equivalence between Case 6.1 and Case 1.7 (for $k = 1$).

⋆⋆⋆ *page 475, Table 6, Cases 6.2 and 6.3, column 5, change both from*

1.1

to

1.2

⋆⋆⋆ *pages 477, 478, 480, 484, 486 & 487, update the following references:*

[**8**] Anderson, I.M., and Kamran, N., The variational bicomplex for second order scalar partial differential equations in the plane, *Duke Math. J.* **87** (1997), 265–319

[**29**] Bryant, R.L., and Griffiths, P.A., Characteristic cohomology of differential systems I, II, *J. Amer. Math. Soc.* **8** (1995), 507–596, *Duke Math. J.* **78** (1995), 531–676.

[**30**] Bryant, R.L., Griffiths, P.A., and Hsu, L.; Hyperbolic exterior differential systems and their conservation laws, Part I, *Selecta Math.* **1** (1995), 21–112.

[**70**] Fels, M., The equivalence problem for systems of second-order ordinary differential equations, *Proc. London Math. Soc.* **71** (1995), 221–240

[**139**] Komrakov, B., Primitive actions and the Sophus Lie problem, *in*: *The Sophus Lie Memorial Conference, Oslo, 1992*, O.A. Laudal and B. Jahren, eds., Scandinavian Univ. Press, Oslo, 1994, pp. 187–269

[**188**] Olver, P.J., Non-associative local Lie groups, *J. Lie Theory* **6** (1996), 23–51.

[**190**] Olver, P.J., Sapiro, G., and Tannenbaum, A., Invariant geometric evolutions of surfaces and volumetric smoothing, *SIAM J. Appl. Math.* **57** (1997), 176–194.

⋆⋆⋆ *page 479, refs* [**37**–**38**]*, change*

Complétes

to

Complètes

⋆⋆⋆ *page 479, refs* [**37**–**42**]*, change*

Gauthiers

to

Gauthier

⋆⋆⋆ *page 483, reference* [**128**]*, change*

dx/dy

to

dy/dx

⋆⋆⋆ *page 500, change*

Galois, E., 3

to

Galois, E., 4

⋆⋆⋆ *page 501, change*

Morikawa, H., 217, [**170**–**172**]

to

Morikawa, H., 217, [**170**]
Morrey, C.B., Jr., 346, [**171**]
Mostow, G.D., 41, 61, [**172**]

⋆⋆⋆ *page 503, add the following to the end of the Author Index.*

Zhitomirskii, M.Y., 31, [**232**], [**233**]

⋆⋆⋆ *page 504, change two entries*

affine-invariant arc length, 339

to

affine-invariant arc length, 241, 339