Math 5587
October 14, 2004

Homework Assignment # 4

Exercises: Strauss pp 118–120: 2, 6.

1. Starting with the Fourier series for the step function $\sigma(x)$, find the Fourier series for the ramp function $\rho(x) = \begin{cases} x, & x > 0, \\ 0, & x < 0. \end{cases}$

2. Find the first and second derivatives (in the context of distributions) of the following functions: (a) $f(x) = \begin{cases} x^2, & 0 < x < 3, \\ x, & -1 < x < 0, \\ 0, & \text{otherwise}, \end{cases}$ (b) $g(x) = e^{-|x|}$.

3. (a) Justify the formula $\delta(2x) = \frac{1}{2}\,\delta(x)$. (b) Is this identity also valid for their Fourier series? Can you explain why or why not? (c) Find a similar formula for $\delta(ax)$ when $a \neq 0$ is a real constant.

4. For each positive integer n, let $g_n(x) = \begin{cases} \frac{1}{2}n, & |x| < 1/n, \\ 0, & \text{otherwise}. \end{cases}$ (a) Sketch a graph of $g_n(x)$. (b) Show that $\lim_{n\to\infty} g_n(x) = \delta(x)$. (c) Evaluate $f_n(x) = \int_{-\infty}^{x} g_n(y)\,dy$ and sketch a graph. Does the sequence $f_n(x)$ converge to the step function $\sigma(x)$ as $n \to \infty$? (d) Find the derivative $h_n(x) = g_n'(x)$. (e) Does the sequence $h_n(x)$ converge to $\delta'(x)$ as $n \to \infty$?

5. Find the Fourier series for the derivative of the delta function $\delta'(x)$: (a) directly from the coefficient formulae; (b) by differentiating the Fourier series for $\delta(x)$. Are you results the same? If not, explain what is going on.

Due: Thursday, October 21

Text: Walter A. Strauss, *Partial Differential Equations: an Introduction*, John Wiley & Sons, New York, 1992.

First Midterm: Thursday, October 28

Will cover sections 1.2, 2.1, 2.2, 3.2, chapter 5, 12.1.

You will be allowed to use one 8” × 11” sheet of notes.