Math 3592H Honors Math I
Quiz 3, Thursday Nov. 3, 2016

Instructions:
15 minutes, closed book and notes, no electronic devices.
There are three problems, worth a total of 20 points.

1. (8 points) Parametrize/describe all solutions to the system

$$\begin{array}{ccccccccc} x & + & y & + & z & + & w & = & 0 \\ x & + & 2y & + & 3z & + & 4w & = & 1. \end{array}$$

2. (4 points) Prove or disprove:
Assume two functions $\mathbf{f}, \mathbf{g} : \mathbb{R}^n \to \mathbb{R}^n$ are both differentiable on $\mathbb{R}^n$, and satisfy $\mathbf{f}(\mathbf{g}(\mathbf{x})) = \mathbf{x}$ and $\mathbf{g}(\mathbf{f}(\mathbf{x})) = \mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^n$.

Then for every $\mathbf{a}$ in $\mathbb{R}^n$, the Jacobian matrix $J\mathbf{f}(\mathbf{a})$ is invertible.

3. (8 points total)
Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 0 & 3 & c \end{bmatrix}.$$

(a) (4 points) Find all values of c that make A invertible

(b) (2 points) For each of the values of c found in part (a), how many solutions are there to the matrix system $A\mathbf{x} = \mathbf{0}$?

(c) (2 points) For each of the values of c found in part (a), how many solutions are there to the matrix system $A\mathbf{x} = \mathbf{e}_1$?