Math 3592H Honors Math I
Quiz 4, Thursday Dec. 1, 2016

Instructions:
20 minutes, closed book and notes, no electronic devices. There is one problem worth 20 points, with four parts each worth 5 points.

1. For any scalar c in $\mathbb{R}$, consider the symmetric matrix $A = \begin{bmatrix} c & 1 & 1 \\ 1 & c & 1 \\ 1 & 1 & c \end{bmatrix}$.

(a) Show that $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector for A. What is its eigenvalue?

(b) What are *all* of the eigenvalues of A?

(c) Find an explicit orthonormal basis $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ for $\mathbb{R}^3$ consisting of eigenvectors for A.

(d) Find an explicit 3×3 matrix P which is orthogonal ($P^{-1} = P^\top$) and for which $P^T AP$ is diagonal.