**Math 3593H Honors Math II**
**Midterm exam 1, Thursday February 16, 2017**

**Instructions:**
50 minutes, closed book and notes, no electronic devices.
There are four problems, worth 25 points each.

1.(i) (10 points) Show the solution set in $\mathbb{R}^3$ to this system is a manifold:

$$x^2 + y^2 = z,$$
$$x + y + z = 4.$$

(ii) (5 points) What is its dimension as a manifold?

(iii) (10 points) Find equations that cut out its tangent space $T_{\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}} M$.

2. (i) (10 points) Compute the $4^{th}$ degree Taylor polynomial $P^4_{f,\left(\begin{smallmatrix}0\\0\end{smallmatrix}\right)}$ at the origin in $\mathbb{R}^2$, for $f\left(\begin{smallmatrix}x\\y\end{smallmatrix}\right) = e^{x^2-3y^2+y^3}$.

(ii) (5 points) Prove that $f$ has a critical point at the origin in $\mathbb{R}^2$.

(iii) (10 points) Classify this critical point as either a local maximum, a local minimum, a saddle, or something indeterminate.

3. Write down a system of $m$ equations in $m$ unknowns, for some value of $m$, whose solution would let you compute the point(s) in $\mathbb{R}^2$ on the hyperbola $xy = 1$ closest to the point $\binom{1}{0}$. Don't bother with solving the system, but do explain what you would do with its solution to find the closest point(s).

4. Find the signature of the quadratic form $\mathbb{R}^4 \xrightarrow{Q} \mathbb{R}$ in the four variables $w, x, y, z$ defined by

$$Q\left(\begin{smallmatrix} w \\ x \\ y \\ z \end{smallmatrix}\right) = \det \begin{bmatrix} x & y \\ z & w \end{bmatrix}.$$