**Math 5711 Combinatorial optimization**
**Spring 2006, Vic Reiner**
**Midterm exam 1- Due Wednesday February 22, in class**

**Instructions:** This is an open book, open library, open notes, open web, take-home exam, but you are *not* allowed to collaborate. The instructor is the only human source you are allowed to consult.

1. Consider the following LP problem:

$$\begin{array}{lrrl} \text{minimize} & & x_2 & \\ \text{subject to} & x_1 & +x_2 & \geq 1 \\ & 3x_1 & +2x_2 & \leq 6 \\ & x_1, & x_2 & \geq 0 \end{array}$$

(a) (5 points) Rewrite this problem in Chvátal's standard form.

(b) (20 points) Solve this problem using the two-phase simplex method. Be sure to write down each dictionary in both Phase I and II, and the entering/leaving variables at each pivot step. You do not need to show the algebra used in rewriting the dictionaries.

2. Consider as primal the LP listed in Chvátal problem 1.4 on page 9.

(a) (5 points) Write down its dual LP.

(b) (5 points) Find necessary and sufficient conditions for the numbers $s$ and $t$ to make the dual LP have an optimal solution, and find this optimal solution (with explanation).

(c) (5 points) Find necessary and sufficient conditions for the numbers $s$ and $t$ to make the dual LP infeasible (with explanation).

(d) (5 points) Find necessary and sufficient conditions for the numbers $s$ and $t$ to make the dual LP unbounded (with explanation).

3. (15 points) Chvátal problem 1.5 on page 10.

4. (15 points) Chvátal problem 3.10 on page 44.

5. (10 points) Your friend tells you that an optimum for the following LP problem

$$\begin{array}{llllll} \text{maximize} & & x_2 & & \\ \text{subject to} & x_1 & +x_2 & +x_3 & \leq 2 \\ & x_1 & & & \leq 1 \\ & x_1, & x_2, & x_3 & \geq 0 \end{array}$$

is achieved by $[x_1^*, x_2^*, x_3^*] = [1, 1, 0]$. Use the complementary slackness equations (or equivalently, Theorem 5.3 of Chvátal) to either prove that your friend is correct, or to prove that they are incorrect.

Do *not* solve LP by some other means as part of your solution, but you are, of course, free to do so as a check for yourself.

6. (15 points) Chvátal problem 9.2 on page 146.